Professor Dr. Stephan Held
Malte Schürks

Exercise Set 12

Exercise 12.1. Let $G = (V, E)$ be a forest with $|V| \geq 2$. Let $p := |\{v \in V : |\delta_G(v)| = 1\}|$ and $q := |\{v \in V : |\delta_G(v)| = 0\}|$. Show that the minimum cardinality of a set $F \subseteq \{\{v, w\} : v, w \in V\}$ of new edges that makes the graph $(V, E \cup F)$ 2-edge-connected is $\lceil \frac{1}{2}p \rceil + q$.

(4 points)

Exercise 12.2. Show that every edge-minimal k-edge-connected graph has two vertices of degree k.

(4 points)

Exercise 12.3. Consider the LP relaxation of the Survivable Network Design Problem described in the lecture. Assume that f is obtained from an instance of the SNDP rather than being an arbitrary proper function. Show that the SNDLP can be formulated as an LP of polynomial size in this case. Specifically, show that there is an LP of polynomial size with variables $(x_e)_{e \in E(G)} \cup X$ such that $x^* \in \mathbb{R}^{E(G)}$ is feasible for the SNDLP if and only if there is $y^* \in \mathbb{R}^X$ such that (x^*, y^*) is feasible for your LP.

(4 points)

Exercise 12.4. Find a 2-factor approximation algorithm for the Point-to-Point Connection Problem: Given an undirected graph G with edge weights $c : E(G) \to \mathbb{R}_+$ and sets $S, T \subseteq V(G)$ with $S \cap T = \emptyset$ and $|S| = |T| \geq 1$, find a set $F \subseteq E(G)$ of minimum cost such that there is a bijection $\pi : S \to T$ and paths from s to $\pi(s)$ for all $s \in S$ in $(V(G), F)$.

Hint: You may use that Jain's algorithm computes a 2-approximation to the SNDIP for arbitrary proper functions f.

(4 points)

Deadline: January 18th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws23/cows23.html`

In case of any questions feel free to contact me at schuerks@dm.uni-bonn.de.