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Exercise Set 8

Exercise 8.1. Consider the metric s-t path TSP: Given an instance of METRIC TSP and two vertices s and t, we look for a Hamiltonian s-t path of minimum weight. Describe a $\frac{5}{3}$-factor approximation algorithm, generalizing Christofides' Algorithm.

(4 points)

Exercise 8.2. Let (G, u, s, t) be a network and $U := \delta^+(s)$. Let

$$P := \Big\{ x \in \mathbb{R}_+^U : \text{there is an } s\text{-}t \text{ flow } f \text{ in } (G,u) \text{ with } f(e) = x_e \text{ for all } e \in U \Big\}.$$

Prove that P is a polymatroid.

(4 points)

Exercise 8.3. Let E be a finite set and $P \subseteq \mathbb{R}^E$ be a polymatroid. Show that there is some submodular set function f with $f(\emptyset) = 0$, f monotone, i.e. $f(X) \leq f(Y)$ for all $X \subseteq Y \subseteq E$ and $P = P(f)$.

(4 points)

Exercise 8.4. Prove that a nonempty compact set $P \subseteq \mathbb{R}_+^n$ is a polymatroid if and only if

(i) For all $0 \leq x \leq y \in P$ we have $x \in P$.

(ii) For all $x \in \mathbb{R}_+^n$ and all $y, z \leq x$ with $y, z \in P$ that are maximal with this property (i.e. $y \leq w \leq x$ and $w \in P$ implies $w = y$, and $z \leq w \leq x$ and $w \in P$ implies $w = z$) we have $\mathbb{1}y = \mathbb{1}z$, where $\mathbb{1}$ is the vector whose entries are all 1.

(4 points)

Deadline: December 7th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws23/cows23.html`

In case of any questions feel free to contact me at schuerks@or.uni-bonn.de.