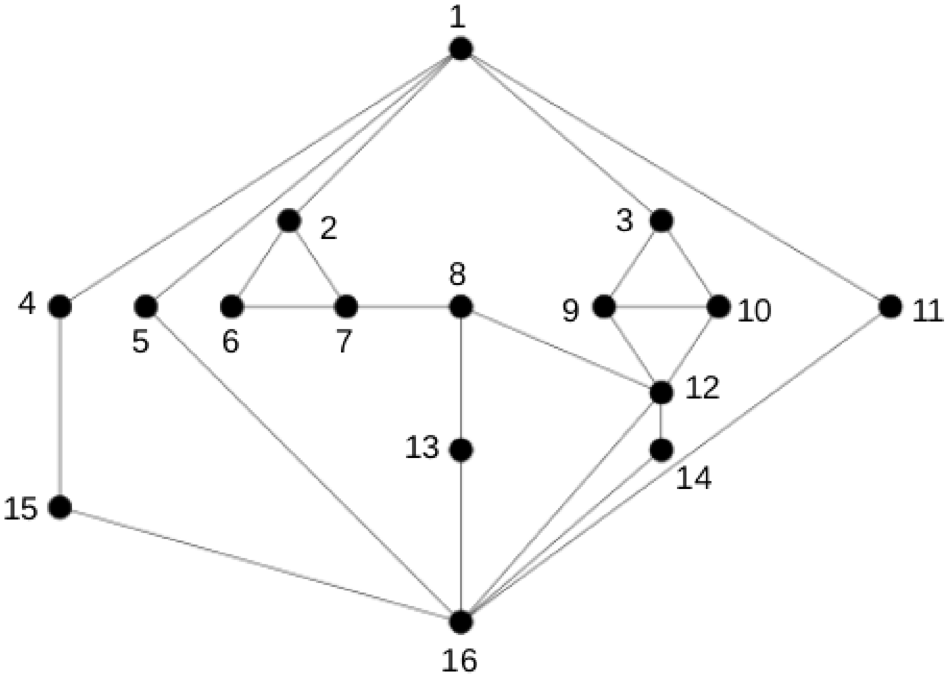
Professor Dr. Stephan Held
Malte Schürks

Exercise Set 4

Figure 4.1: Graph for exercise 4.1



Exercise 4.1. Find a maximum matching in the graph in Figure 4.1. Prove that the matching is indeed maximum, e.g. using the Berge-Tutte-formula.

(3 points)

Exercise 4.2. Recall Exercise 3.2. Although you only had to solve parts (vi) and (vii) for bipartite graphs, they can also be solved in general for undirected graphs (with the same runtime), only with a more complicated algorithm in part (vi). Using this, describe a linear-time approximation scheme for the Maximum Cardinality Matching Problem. More specifically, describe an algorithm which takes as input an undirected graph G and a positive number ε, outputs a matching M in G such that $|M| \geq (1-\varepsilon) \cdot \nu(G)$, and runs in

$$O\left(\frac{1}{\varepsilon} \cdot (|V(G)| + |E(G)|)\right)\text{-time.}$$

(4 points)

Exercise 4.3. Let G be a 3-regular undirected graph.

(a) Assume G is simple. Show that there is a matching in G covering at least $(7/8) \cdot |V(G)|$ vertices.

(b) Give an example to prove that the bound of item (a) is tight.

(c) Show that the assumption that G is simple in item (a) is necessary.

(3 + 1 + 1 points)

Exercise 4.4. Let G be any graph and denote by Y, X, W its Gallai-Edmonds decomposition. Let $v \in X$ and denote by Y', X', W' the Gallai-Edmonds decomposition of $G - v$. Show that $Y' = Y$, $X' = X \backslash \{v\}$ and $W' = W$.

(4 points)

Deadline: November 9^{nd}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws23/cows23.html`

In case of any questions feel free to contact me at schuerks@or.uni-bonn.de.