Professor Dr. Stephan Held
Malte Schürks

Exercise Set 2

Exercise 2.1. Prove that the number of ears in any two odd ear-decompositions of a factor-critical graph G is the same.

(3 points)

Exercise 2.2. A set of students applies for a set of seminars. Each student chooses exactly three seminars. Two seminars are chosen by 40 students, all others by fewer.

(a) Prove that each student can be assigned to a seminar they chose without assigning more than 13 students to any seminar.

(b) Show how to compute such an assignment in $\mathcal{O}(n^2)$ time, where n is the number of seminars.

(3+1 points)

Exercise 2.3. Show that a graph G is factor-critical if and only if G is connected and for every vertex $v \in V(G)$ we have $\nu(G - v) = \nu(G)$.

(4 points)

Exercise 2.4. Let G be a graph, $n := |V(G)|$ even, and for any set $X \subseteq V(G)$ with $|X| \leq \frac{3}{4}n$ we have

$$\left| \bigcup_{x \in X} \Gamma(x) \right| \geq \frac{4}{3}|X|.$$

Prove that G has a perfect matching.

Hint: Let S be a set violating the Tutte condition. Prove that the number of connected components in $G - S$ with just one element is at most $\max\left\{0, \frac{4}{3}|S| - \frac{1}{3}n\right\}$. Consider the cases $|S| \geq \frac{n}{4}$ and $|S| < \frac{n}{4}$ separately.

(5 points)

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Deadline: October 26th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws23/cows23.html`

In case of any questions feel free to contact me at schuerks@or.uni-bonn.de.