Professor Dr. Stephan Held
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Exercise Set 6

Exercise 6.1. An *odd cover* for a graph G is a set $F \subseteq E(G)$ such that if we successively contract in G the elements of F (and delete possible loops), the resulting graph is Eulerian. Note that contractions may lead to parallel edges. Consider the Minimum Weight Odd Cover Problem: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an odd cover with minimum weight (or show that G has no odd cover). Show that this problem can be solved in polynomial time.

(5 points)

Exercise 6.2. The Undirected Minimum Mean-Weight Cycle Problem is the following: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}$, find a cycle C whose mean-weight $c(E(C))/|E(C)|$ is minimum, or determine that G is acyclic. Consider the following algorithm for the Undirected Minimum Mean-Weight Cycle Problem: First determine with a linear search whether G has cycles or not, and if not return with this information. Let $\gamma := \max\{c(e) : e \in E(G)\}$ and define a new edge-weight function via $c'(e) := c(e) - \gamma$. Let $T := \emptyset$. Now iterate the following: Find a minimum c'-weight T-join J with a polynomial (black-box) algorithm. If $c'(J) = 0$, return any zero-c'-weight cycle. Otherwise, let $\gamma' := c'(J)/|J|$, reset c' via $c'(e) \leftarrow c'(e) - \gamma'$, and continue.

Show that this algorithm works correctly and runs in polynomial time. Also, explain how to the get the cycle to be returned in the case $c'(J) = 0$.

(6 points)

Exercise 6.3. Consider the Directed Chinese Postman Problem: Given a strongly connected simple digraph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find a function $f : E(G) \to \mathbb{N} \setminus \{0\}$ such that if each edge $e \in E(G)$ is replaced by $f(e)$ copies of itself, the resulting graph is Eulerian, and such that f minimizes $\sum_{e \in E(G)} f(e)c(e)$ among functions with this property. Show that this problem can be linearly reduced to the Minimum Cost Integral Flow Problem (i.e. the Minimum Cost Flow Problem with the additional requirement that the flow must be integral).

(4 points)

Exercise 6.4. Let G be a graph and $T \subseteq V(G)$ with $|T|$ even. Prove:

(i) A set $F \subseteq E(G)$ intersects every T-join if and only if it contains a T-cut.

(ii) A set $F \subseteq E(G)$ intersects every T-cut if and only if it contains a T-join.

(5 points)

Deadline: November 21st, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html`

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.

Special announcement: The student council of mathematics will organize the Maths Party on 28/11 at the N8schicht. The presale will be held on Tue 26/11, Wed 27/11 and Thu 28/11 in the mensa Poppelsdorf. Further information can be found at `fsmath.uni-bonn.de`.]