

Exercise Set 4

Exercise 4.1. Let $S = \{1, \dots, n\}$ for some $n \geq 1$.

- (i) Suppose $0 \leq k \leq n - 1$ and consider the bipartite graph $G = (A \dot{\cup} B, E)$ where

$$\begin{aligned} A &:= \{X \subseteq S : |X| = k\}, \\ B &:= \{Y \subseteq S : |Y| = k + 1\}, \\ E &:= \{\{X, Y\} : X \in A, Y \in B, X \subseteq Y\}. \end{aligned}$$

Show that there is a matching covering A if $k < n/2$, and that there is a matching covering B if $k > n/2 - 1$.

- (ii) Suppose \mathcal{F} is a family of subsets of S with the property that no element of \mathcal{F} is contained in another element of \mathcal{F} . Show that:

$$|\mathcal{F}| \leq \binom{n}{\lfloor \frac{n}{2} \rfloor}$$

and that this bound is tight (for every n).

(1+3 points)

Exercise 4.2. Consider the BOTTLENECK MATCHING PROBLEM: Given an undirected graph G with edge weights $c : E(G) \rightarrow \mathbb{R}$, find a perfect matching M (if one exists) minimizing $\max\{c(e) : e \in M\}$.

Show how to solve the BOTTLENECK MATCHING PROBLEM in $O(nm \log n)$ time using a modified variant of Edmonds' (unweighted) PERFECT MATCHING algorithm.

(4 points)

Exercise 4.3. Let G be a graph with edge weights $c : E(G) \rightarrow \mathbb{R}$ and let M be a matching in G with $|M| = k$ that has minimum weight among all matchings in G that contain exactly k edges. Let P be an M -augmenting path in G with minimum gain. Let $M' := M \triangle E(P)$. Prove that M' has minimum weight among all matchings in G that contain exactly $k + 1$ edges.

(4 points)

Exercise 4.4. Let G be a graph with edge weights $c : E(G) \rightarrow \mathbb{R}_{\geq 0}$. Let $\nu(G, c)$ denote the maximum weight of a matching in G . Suppose M is a matching in G for which there is no 2-augmentation with strictly positive gain. Show that

$$c(M) \geq \frac{2}{3} \nu(G, c).$$

(4 points)

Deadline: November 9th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.