Professor Dr. Stefan Hougardy
Jannik Silvanus

Exercise Set 3

Exercise 3.1. Let G be a k-vertex-connected graph which has neither a perfect nor a near-perfect matching.

(i) Show that $\nu(G) \geq k$.

(ii) Show that $\tau(G) \leq 2 \cdot \nu(G) - k$.

(2+2 points)

Exercise 3.2. Given a bipartite graph G and edge weights $c : E(G) \to \mathbb{R}_{\geq 0}$, we consider the iterations of the HUNGARIAN METHOD on (G, c). Denote by k the number of iterations, and for $1 \leq i \leq k$, denote by G'_i the spanning subgraph of G given by the edges satisfying $c(\{x, y\}) = w(x) + w(y)$ in the i-th iteration.
Prove or disprove: For all $1 \leq i < k$, we have $E(G'_i) \subseteq E(G'_{i+1})$.

(4 points)

Exercise 3.3. Consider the MINIMUM COST EDGE COVER PROBLEM: Given a graph G with weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. Show that the MINIMUM COST EDGE COVER PROBLEM can be linearly reduced to the MINIMUM WEIGHT PERFECT MATCHING PROBLEM.

(4 points)

Exercise 3.4. Consider the SHORTEST EVEN/ODD PATH PROBLEM: Given a graph G with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an s-t-path P of even/odd length in G that minimizes $\sum_{e \in E(P)} c(e)$ among all s-t-paths of even/odd length in G. Show that both the even and the odd version can be linearly reduced to the MINIMUM WEIGHT PERFECT MATCHING PROBLEM.

(4 points)

Deadline: November 2^{nd}, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.