Winter term 2013/14 Research Institute for Discrete Mathematics
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Combinatorial Optimization

Exercise Sheet 12

Exercise 12.1:
Let $G = (V, E)$ be a graph. We define $\mathcal{F} := \{X \subseteq V \mid X \text{ is covered by some matching}\}$ and $\mathcal{F}^* := \{X \subseteq V \mid X \text{ is exposed by some maximum matching}\}$

1. Show that $(V, \mathcal{F})$ and $(V, \mathcal{F}^*)$ are matroids.
2. Show that $(V, \mathcal{F}^*)$ is the dual matroid of $(V, \mathcal{F})$.

(2+2 points)

Exercise 12.2:
Let $\mathcal{M}_1, \mathcal{M}_2$ be matroids on E. Let B be a maximal partitionable subset with respect to $\mathcal{M}_1$ and $\mathcal{M}_2^*$, and let J_1, J_2 be an associated partitioning. Furthermore, let B_2 be a basis of $\mathcal{M}_2^*$ with $J_2 \subseteq B_2$. Show that $B \setminus B_2$ is a common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ of maximum cardinality.

(4 points)

Exercise 12.3:
Let G be a connected graph and k an integer. Prove:

(i) G has two edge-disjoint spanning trees if and only if there does not exist a partition of V into sets $V_0, \ldots, V_p$ such that $|E(V_0, \ldots, V_p)| < 2p$, where $E(V_0, \ldots, V_p)$ denotes the set of edges with endpoints in different V_i.

(ii) G has k edge-disjoint spanning trees if and only if there does not exist a partition of V into sets $V_0, \ldots, V_p$ such that $|E(V_0, \ldots, V_p)| < kp$.

(3+3 points)

Exercise 12.4:
Let $(E, \mathcal{F})$ be a matroid, $A \in \mathcal{F}$ arbitrary, and $\mathcal{F}_A := \{X \Delta A \mid X \in \mathcal{F}\}$. Prove that $(E, \mathcal{F}_A)$ is a greedoid.

(2 points)

Deadline: Thursday, January 30, 2014, before the lecture.