Winter term 2013/14
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Combinatorial Optimization

Exercise Sheet 9

Exercise 9.1:
Let G be an undirected graph.

(i) Show that the cone generated by (i.e. the set of non-negative linear combinations of) the incidence vectors of all circuits in G, called the circuit cone of G, is determined by

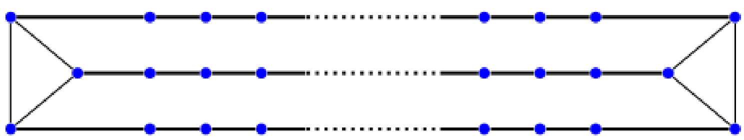
$$x_e \geq 0 \qquad \text{for each } e \in E(G)$$
$$\sum_{f \in F} x_f \geq 2x_e \qquad \text{for each cut } F \subseteq E(G) \text{ and } e \in F.$$

(ii) Show that the separation problem for the circuit cone can be solved in polynomial time.

Hint: Use exercise 8.4.

(4 points)

Exercise 9.2:
Find an optimum tour of the metric closure of the following graph G and prove its optimality. Derive that $\frac{4}{3}$ is a lower bound for the integrality ratio of the subtour polytope.



(4 points)

Exercise 9.3:
Let T be an optimum tour for an instance (K_n, c) of the Metric Tsp and let T' be a shortest tour different from T. Show that

$$\frac{c(T') - c(T)}{c(T)} \leq \frac{2}{n}$$

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(4 points)

Exercise 9.4:
Let G be an undirected graph. A Hamiltonian circuit in G is a circuit in G containing all vertices of G. Prove that $x \in \mathbb{Z}^{E(G)}$ is the incidence vector of a Hamiltonian circuit in G if and only if it satisfies the following constraints:

$$0 \le x_e \le 1 \qquad (e \in E(G)) \tag{1}$$

$$\sum_{e \in \delta(v)} x_e = 2 \qquad (v \in V(G)) \tag{2}$$

$$\sum_{e \in E(G[X])} x_e \le |X| - 1 \qquad (\emptyset \ne X \subset V(G)) \tag{3}$$

$$\sum_{e \in E(G[Y]) \cup F} x_e \le |Y| + \frac{|F| - 1}{2} \qquad (Y \subseteq V(G), F \subseteq \delta(Y), |F| \text{ odd}) \tag{4}$$

(5)

Show that the Separation Problem for these constraints can be solved in polynomial time.

(4 points)

Deadline: Thursday, December 19, 2013, before the lecture.