

Combinatorial Optimization

Exercise Sheet 6

Exercise 6.1:

Show how to solve the MAXIMUM WEIGHT b -MATCHING PROBLEM for the special case where $b(v)$ is even for all vertices v in strongly polynomial time.

Hint: Reduction to the MINIMUM COST FLOW PROBLEM.

(4 points)

Exercise 6.2:

Show that a minimum-weight perfect simple 2-matching in an undirected graph G can be found in $O(|V(G)|^6)$ time (you may use that minimum weight perfect matchings can be computed in $O(|V(G)|^3)$).

(4 points)

Exercise 6.3:

Let G be a graph with edge weights $c : E(G) \rightarrow \mathbb{R}_{>0}$. A set $F \subseteq E(G)$ is called *odd cover* if the graph which results from G by successively contracting each $e \in F$ is Eulerian. Show that it is possible in polynomial time to find an odd cover F that minimizes $c(F)$ or to decide that none exists.

(4 points)

Exercise 6.4:

Consider the MAXIMUM WEIGHT CUT PROBLEM, i.e. given a graph G and edge weights $c : E(G) \rightarrow \mathbb{R}_{>0}$, find a cut $E' \subset E(G)$ with maximum weight $c(E')$. The problem is NP -hard even for $c \equiv 1$.

Show that this problem can be solved in polynomial time if G is planar.

Hint: Use the fact that a connected planar graph is bipartite if and only if its planar dual is Eulerian.

(4 points)

Deadline: Thursday, November 28, 2013, before the lecture.