Winter term 2013/14
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Combinatorial Optimization

Exercise Sheet 3

Exercise 3.1:
Prove: A graph G has a perfect matching if and only if for each $X \subseteq V(G)$, the graph $G - X$ has at most $|X|$ factor-critical components.

(4 points)

Exercise 3.2:
Let G be a bipartite graph with edge weights $c : E(G) \to \mathbb{R}$ and let M_k be a matching in G with $|M| = k$ that has minimum weight among all matchings in G that contain exactly k edges. Let p be an augmenting path in G with respect to M_k with minimum weight. Let $M_{k+1} := M \triangle E(p)$. Prove that $|M_{k+1}| = k + 1$ and M_{k+1} has minimum weight among all matchings in G that contain exactly $k + 1$ edges.

(4 points)

Exercise 3.3:
Let G be an undirected graph with edge weights $c : E(G) \to \mathbb{R}_{>0}$, and let $v, w \in V$ be two distinct vertices. Describe a polynomial-time algorithm that computes, among all v-w-paths having an even number of edges, a path of minimum weight, and prove its correctness.
Hint: Use that minimum-weight perfect matchings can be computed in polynomial time.

(4 points)

Exercise 3.4:
Show how the following problem can be solved in polynomial time: Given a graph G and edge weights $c : E(G) \to \mathbb{R}_{>0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$.

(4 points)

Deadline: Thursday, November 7, 2013, before the lecture.