Winter term 2013/14 Research Institute for Discrete Mathematics
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Combinatorial Optimization

Exercise Sheet 2

Exercise 2.1:
Prove that an undirected graph G is factor-critical if and only if G is connected and $\nu(G) = \nu(G - v)$ for all $v \in V(G)$.

(4 points)

Exercise 2.2:
Let $G = (V, E)$ be a graph. A set $\mathcal{H} = \{S_1, \dots, S_k, v_1, \dots v_r\}$ has property A if

- $S_i \subseteq V$ and $|S_i|$ is odd for $1 \leq i \leq k$,
- $v_i \in V$ for $1 \leq i \leq r$, and
- for each $e \in E$ either $e \subseteq S_i$ for some $i \in \{1, \dots, k\}$ or $v_i \in e$ for some $i \in \{1, \dots, r\}$.

The weight of a set $\mathcal{H}$ with property A is $w(\mathcal{H}) := r + \sum_{i=1}^{k} \frac{|S_i|-1}{2}$. Prove

$$\nu(G) = \min\{w(\mathcal{H}) \,|\, \mathcal{H} \text{ is a set with property A}\}.$$

(4 points)

Exercise 2.3:
Let $G = (V, E)$ a graph and $X \subseteq V$. Let $\beta(G, X)$ be the maximum size of a set $Y \subseteq X$ for which there is a matching in G that covers Y. Prove

$$\beta(G, X) = \min_{U \subseteq V} |X| + |U| - q_X(U).$$

Here $q_X(U)$ denotes the number of odd connected components of $G - U$ whose vertices are all in X.
Hint: Construct a new graph by adding vertices V' with $|V'| = |V|$ and all edges between vertices in V'. For some $X' \subseteq V'$ with $|X'| = |X| - \beta(G, X)$ add the edges $E(X', V)$ and $E(V' \setminus X', V \setminus X)$.

(4 points)

Exercise 2.4:
Let $G = (V, E)$ be a k-vertex-connected graph with $2\nu(G) < |V| - 1$.

- Prove $\nu(G) \geq k$.
- Prove $\tau(G) \leq 2\nu(G) - k$.

(4 points)

Deadline: Thursday, October 31, 2013, before the lecture.