Winter term 2013/14 Research Institute for Discrete Mathematics
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Combinatorial Optimization

Exercise Sheet 1

Exercise 1.1:
Find an infinite counterexample to Hall's Theorem. More precisely: Find a bipartite graph $G = (A \dot{\cup} B, E)$ with $A = \mathbb{N}$, $B = \mathbb{N}$ and $|N(S)| \geq |S|$ for every $S \subseteq A$ and every $S \subseteq B$, such that G contains *no* perfect matching.

(4 points)

Exercise 1.2:
Let $G = (V, E)$ be a bipartite graph with bipartition $V = A \dot{\cup} B$. Prove that there exists a forest $F = (V, E')$ with $E' \subseteq E$ and $|\delta_F(v)| = 2$ for all $v \in A$ if and only if $|N_G(X)| \geq |X| + 1$ holds for every non-empty $X \subseteq A$.

(4 points)

Exercise 1.3:
Let G be a bipartite graph. For each $v \in V(G)$ let $<_v$ be a linear ordering of $\delta(v)$. Prove that there is a matching $M \subset E(G)$ such that for each $e \in E(G) \setminus M$ there is an edge $f \in M$ and a vertex $v \in V(G)$ such that $v \in (e \cap f)$ and $e <_v f$.

(4 points)

Exercise 1.4:
Prove that in every 3-regular graph there is a matching that covers at least 7/8 of all vertices.

(4 points)

Deadline: Thursday, October 24, 2013, before the lecture.
Note: Solutions may be submitted in groups of at most two students (groups of exactly two students are strongly recommended). Every student needs to achieve at least 50% of the sum of the possible points of all exercises.