Prof. Dr. Stefan Hougardy
Markus Struzyna

Exercises 6

Exercise 1:

An *odd set cover* of a graph G is a set $\mathcal{H} = \{S_1, \ldots, S_k, v_1, \ldots, v_r\}$ of subsets $S_i \subseteq V(G)$ of odd cardinality and vertices $v_i \in V(G)$ such that for each edge $e \in E(G)$ either both endpoints of e are contained in one of the S_i's or e is incident to one of the v_i's.
The *weight* of an odd set cover $\mathcal{H}$ is $w(\mathcal{H}) := r + \sum_{i=1}^{k} \frac{|S_i|-1}{2}$.
Prove the following generalization of König's Theorem for general graphs:

$$\nu(G) = \min\{w(\mathcal{H}) \mid \mathcal{H} \text{ odd set cover of G}\} \text{ for any graph } G.$$

(4 points)

Exercise 2:
Let $G = (V, E)$ be a graph, $c : E \to \mathbb{R}$ and M a matching in G with $c(M) > 0$. Consider an M-augmenting path (or cycle) P and its **relative gain** $\text{gain}_{rel}(P) := \frac{c(M \Delta P)}{c(M)}$. Recall that $\text{aug}(v)$ (the maximum gain 2-augmentation centered at v) can be obtained in linear time in $\deg(v) + \deg(\mu(v))$. What is the fastest algorithm to compute a maximum **relative** gain 2-augmentation centered at v?

(4 points)

Exercise 3:
Let G be a graph. Show that a minimum edge cover in G can be computed in polynomial time.

(4 points)

Deadline: Tuesday, November 23rd, before the lecture.