

## Exercises 5

**Exercise 1:**

Show: the number of ears in any two odd ear-decompositions of a factor-critical graph  $G$  is the same.

(4 points)

**Exercise 2:**

Prove that a minimal factor-critical graph  $G$  has at most  $\frac{3}{2}(|V(G)| - 1)$  edges and this bound is tight.

(4 points)

**Exercise 3:**

Let  $G$  be a graph,  $M$  a maximum matching in  $G$  and  $F$  as well as  $F'$  two special blossom forests w.r.t  $M$ , each with the maximum possible number of edges. Show that the set of **inner** vertices in  $F$  and  $F'$  is the same.

(4 points)

**Exercise 4:**

Let  $G$  be a  $k$ -connected graph with  $2\nu(G) < |V(G)| - 1$ . Prove:

- a.  $\nu(G) \geq k$ ,
- b.  $\tau(G) \leq 2\nu(G) - k$ .

(Use the Gallai-Edmonds Theorem)

(4 points)

**Deadline:** Tuesday, November 16th, before the lecture.