Combinatorial Optimization
Winter term 2010/2011

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# Exercises 10

**Exercise 1:**
Compute (sharp) lower bounds for the rank quotients of the following independence systems $(E, \mathcal{F})$:

(a) $E = V(G)$ and $\mathcal{F}$ stable sets in $G$.

(b) $E = E(G)$ and $\mathcal{F}$ are subsets of a Hamiltonian circle in G.

(c) $E = E(G)$ and $\mathcal{F}$ are subsets of an $s-t$-path with $s, t \in V(G)$ and $t$ reachable from $s$.

(d) $E = \{1, \dots, n\}$ and non-negative weights $w_j,\ j = 1, \dots, n$ and $k \in \mathbb{R}_+$. $\mathcal{F}$ are subsets of total weight $\leq k$.

(e) $E = E(G)$, $\mathcal{F}$ are subsets of Steiner trees in $G$.

(f) $E = E(G)$, $\mathcal{F}$ are subsets of branchings in $G$.

(g) $E = E(G)$, $\mathcal{F}$ contains matchings in $G$.

(6 points)

**Exercise 2:**
Show that the following decision problem is $\mathcal{NP}$-complete: Given three matroids $(E, \mathcal{F}_1)$, $(E, \mathcal{F}_2)$, $(E, \mathcal{F}_3)$ (by some oracle) and a $k \in \mathbb{N}$. Exists an $F$ in $\mathcal{F}_1 \cap \mathcal{F}_2 \cap \mathcal{F}_3$ such that $|F| \geq k$ ?

(3 points)

**Exercise 3:**
Let $\mathcal{M}$ be a graphic matroid. Prove that there exists a connected graph $G$ such that

$$\mathcal{M} \cong \mathcal{M}(G)$$

(4 points)

**Exercise 4:**
Let $G$ be an undirected graph, $k \in \mathbb{N}$ and let $\mathcal{F} := \{F \subseteq E(G) | F \text{ is union of } k \text{ forests }\}$.
Prove: $(E(G), \mathcal{F})$ is a matroid.

(3 points)

**Deadline:** Tuesday, December 21st, before the lecture.