Summer term 2026
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Linear and Integer Optimization

Exercise Sheet 11

Exercise 11.1: Show that each unimodular square matrix arises from the identity matrix by a series of elementary unimodular column operations. (5 Points)

Exercise 11.2: Show that $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is not totally unimodular but $\{x \in \mathbb{R}^3 \mid Ax = b\}$ is integral for all integral vectors b. (4 points)

Exercise 11.3: Let $A \in \{0,1\}^{m \times n}$ be a matrix where in each column the 1's are arranged consecutively, i.e. for each column $j \in \{1, \dots, n\}$ there are $i_1^j, i_2^j \in \{1, \dots, m\}$ s.t.:

$$a_{ij} = \begin{cases} 1, & i_1^j \leq i \leq i_2^j \\ 0, & \text{else} \end{cases}$$

for $j \in \{1, \dots, n\}$ and $i \in \{1, \dots, m\}$ (if $i_1^j > i_2^j$, the column consists of zeros only). Show that A is totally unimodular. (5 Points)

Exercise 11.4: Consider the following capacitated facility location problem: given a set of clients C and a set of potential facility locations F, a metric ℓ on $C \cup F$ representing connection costs, facility opening costs $p : F \to \mathbb{R}_{\geq 0}$ and capacities $c : F \to \mathbb{N}$, and client demands $d : C \to \mathbb{N}$, find a set $I \subseteq F$ of facilities to be opened and an assignment $f : C \to I$ of clients to open facilities such that the capacity bounds are respected ($\sum_{c \in f^{-1}(x)} d(c) \leq c(x)$ for all $x \in I$) and the sum of opening costs and connection costs of clients to their assigned facilities is minimized.

1. Model this problem as an integer program.
2. Give a non-trivial example instance for which the LP relaxation of your IP has a unique optimum solution which is integral. Give an example instance for which every optimal solution of the LP relaxation is fractional. (3+3 Points)

Submission deadline: Tuesday, July 7, 2026, 16:00, via eCampus (in groups of at most 3 students).