Summer term 2026
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Linear and Integer Optimization

Exercise Sheet 10

Exercise 10.1: Prove the following statements.

(a) If $X \subseteq \mathbb{R}^n$ is a finite set of vectors and $c \in \text{cone}(X)$ then there are linearly independent vectors $a_1, \dots, a_k \in X$ such that $c \in \text{cone}(\{a_1, \dots, a_k\})$.

(b) For any pointed rational polyhedral cone $C \subset \mathbb{R}^n$, any Hilbert basis $\{a_1, \dots, a_l\}$ of C and any integral point $x \in C \cap \mathbb{Z}^n$, there are $2n - 1$ vectors from the Hilbert basis such that x is a non-negative integral combination of these vectors. (4+4 points)

Exercise 10.2: Give a set of linear inequalities describing P_I for the following set P:

$$P = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 \neq x_2, x_1 \neq x_3, x_2 \neq x_3, \text{ and } \forall i \in \{1, 2, 3\} : 1 \leq x_i \leq 3\}.$$

(4 Points)

Exercise 10.3: Let $a = (a_1, \dots, a_n) \in (\mathbb{N} \setminus \{0\})^n$ be a vector and β a rational number. Prove that $a^t x \leq \beta$ is TDI if and only if the greatest common divisor of $a_1, \dots, a_n$ is 1. (2 Points)

Exercise 10.4:

(a) Show that the systems

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

define the same polyhedron. Prove that the first system is TDI while the second one is not TDI.

(b) Prove or disprove the following statement: If $Ax \leq b$ (with $A \in \mathbb{Q}^{m \times n}$ and $b \in \mathbb{Q}^m$) is TDI and $\alpha \in \mathbb{Q}_{>0}$, then $\alpha A x \leq \alpha b$ is TDI. (3+3 Points)

Submission deadline: Tuesday, June 30, 2026, 16:00, via eCampus (in groups of at most 3 students).