Summer term 2026
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Linear and Integer Optimization

Exercise Sheet 9

Exercise 9.1: Show that in the inequality

$$\max\{c^t x \mid Ax \le bx \in \mathbb{Z}^n\} \le \min\{b^t y \mid A^t y = c, y \ge 0, y \in \mathbb{Z}^m\}$$

in general, equality does not hold, even if the two corresponding optimization problems are feasible and bounded. (2 Points)

Exercise 9.2: Give an example each of

1. a full-dimensional unbounded rational polyhedron P such that P_I is empty.
2. an unbounded polyhedron P such that P_I is non-empty and bounded.
3. a polyhedron P such that $P_I \neq \emptyset$ is not closed.
4. a feasible and bounded ILP without optimum solution. (2+2+2+2 Points)

Exercise 9.3: Let $P, Q \subseteq \mathbb{R}^n$ be two polyhedra. Show that $P_I + Q_I \subseteq (P+Q)_I$. Give an example where $P_I + Q_I \neq (P+Q)_I$. (5 points)

Exercise 9.4: The Maximum-Stable-Set-Problem is defined as follows. Given a graph G, we are looking for a vertex set $S \subseteq V(G)$ of maximum cardinality $|S|$ such that $\{v, w\} \notin E(G)$ for all $v, w \in S$. A vertex set S with $E(G[S]) = \emptyset$ is called *stable set*. This problem can be modeled by the following ILP:

$$\max \sum_{v \in V(G)} x_v \quad (1)$$

$$s.d. \quad x_v + x_w \le 1 \qquad \forall \{v, w\} \in E(G) \quad (2)$$

$$x_v \in \{0, 1\} \qquad \forall v \in V(G) \quad (3)$$

1. Prove that the integrality gap of the LP relaxation is at least $\frac{|V(G)|}{2}$.
2. Show that following inequalities are valid for the Maximum-Stable-Set-Problem

$$\sum_{v \in V(H)} x_v \le \left\lceil \frac{|V(H)| - 1}{2} \right\rceil \quad \text{for a circuit } H \subseteq G.$$

Give examples, where the optima of the linear relaxations are reduced (strictly). (2+3 Points)

Submission deadline: Tuesday, June 23, 2026, 16:00, via eCampus (in groups of at most 3 students).