Summer term 2026 Research Institute for Discrete Mathematics
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Linear and Integer Optimization

Exercise Sheet 8

Exercise 8.1: Consider the following optimization problem:

$$\begin{array}{rl} \min & \frac{c^t x + d}{f^t x + g} \\ \text{s.t.} & Ax \leq b \\ & \|x\|_\infty \leq R \end{array}$$

where $c, f \in \mathbb{Q}^n$, $d, g, R \in \mathbb{Q}$, $A \in \mathbb{Q}^{m \times n}$, $b \in \mathbb{Q}^m$. You may assume that $f^t x + g > 0$ and $c^t x + d > 0$ for any $x \in \mathbb{R}^n$ with $\|x\|_\infty \leq R$ and that there is a feasible solution. Show that for any $\epsilon > 0$ there is a polynomial-time algorithm computing a feasible solution x^* with $\frac{c^t x^* + d}{f^t x^* + g} \leq \mathrm{OPT}(1 + \epsilon)$ where OPT is the value of an optimum solution. (5 Points)

Exercise 8.2: Let $K \subseteq \mathbb{R}^n$ be a convex set with $B(0, r) \subseteq K \subseteq B(0, R)$ for some numbers $0 < r \leq \frac{R}{2}$.
Assume that you are given an oracle with polynomial running time that computes an optimum solution in K for any linear objective function. Show that there is a separation oracle with polynomial running time for $K^* := \{y \in \mathbb{R}^n \mid y^t x \leq 1 \text{ for all } x \in K\}$. (5 Points)

Exercise 8.3: Use the Ellipsoid Algorithm to show that a given feasible and bounded linear program $\max\{c^t x \mid Ax \leq b\}$ with $A \in \mathbb{Q}^{m \times n}$, $b \in \mathbb{Q}^m$ and $c \in \mathbb{Q}^n$ can be solved in time $O((m + n)^9 (\mathrm{size}(A) + \mathrm{size}(b) + \mathrm{size}(c))^2)$. You may assume that arithmetic operations require a running time linear in the number of bits of the corresponding numbers. (5 points)

Exercise 8.4: Show that there is a polynomial algorithm that computes, given a feasible and bounded LP $\max\{c^t x \in \mathbb{R}^n \mid Ax \leq b\}$ (with $c \in \mathbb{Q}^n$, $A \in \mathbb{Q}^{m \times n}$ and $b \in \mathbb{Q}^m$) such that $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ is pointed, an optimum solution of the LP that is a vertex of P. (5 Points)

Submission deadline: Tuesday, June 16, 2026, 16:00, via eCampus (in groups of at most 3 students).