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Exercise Set 11

Exercise 11.1. Let G be an undirected graph with resistances and capacitances $\mathrm{res}, \mathrm{cap}\colon E \to \mathbb{R}_{>0}$ on the edges. Let Y be a Steiner tree on the terminal set N rooted at the source pin p and sink pins $N \setminus \{p\}$, so that all sink pins are leaves. We interpret Y as an arborescence. Let $\pi\colon N \setminus \{p\} \to \mathbb{R}_{>0}$ be sink capacitances. We denote by $Y[p,q]$ the unique path from p to q in Y.

We define the Elmore delay from p to a sink $q \in N \setminus \{p\}$ in Y as

$$\mathrm{Elmore}_Y(p,q) := \sum_{e=(v,w)\in Y[p,q]} \mathrm{res}(e)\left(\frac{\mathrm{cap}(e)}{2} + \mathrm{downcap}(w)\right)$$

where $\mathrm{downcap}(q)$ is $\pi(q)$ if q is a sink pin and

$$\mathrm{downcap}(w) := \sum_{e=(w,x)\in\delta^+_Y(w)} \left(\mathrm{cap}(e) + \mathrm{downcap}(x)\right)$$

otherwise.

Show that subdividing an edge $e = \{x,y\} \in E(G)$ into edges $e_1 = \{x,z\}$ and $e_2 = \{z,y\}$ using a new vertex z with $\mathrm{cap}(e_1) + \mathrm{cap}(e_2) = \mathrm{cap}(e)$, $\mathrm{res}(e_1) + \mathrm{res}(e_2) = \mathrm{res}(e)$ and $\frac{\mathrm{res}(e_1)}{\mathrm{cap}(e_1)} = \frac{\mathrm{res}(e_2)}{\mathrm{cap}(e_2)}$ does not change the Elmore delay.

(5 points)

Exercise 11.2. Let G be an undirected graph with weights, resistances and capacitances $w, \mathrm{res}, \mathrm{cap}\colon E \to \mathbb{R}_{>0}$ on the edges. Let $\alpha > 0$. For a path P from a to b in G let

$$\mathrm{cost}(P) := w(P) + \alpha\,\mathrm{Elmore}_P(a,b)\ ,$$

where P is interpreted as Steiner tree with two terminals rooted at a and $\pi(b) = 1$.

Let $s,t \in V(G)$. Assume that $\mathrm{cap}(e) = 1$ for all $e \in E(G)$.

Show that one can in polynomial time compute a path P from s to t with minimal cost.

Hint: Use a variation of Dijkstra's algorithm starting from t with labels $(v, \mathrm{downcap}(v))$ for vertices v.

(5 points)

Exercise 11.3. Show that given an initial Steiner tree and shortest r-t paths for all $t \in T$, the remaining steps (cut and reconnect) of the algorithm by Khazraei and Held for the uniform Cost-Distance Steiner tree problem (section 6.3) can be implemented deterministically with the same guarantees in linear time.

(5 points, for polynomial time 3 points)

Exercise 11.4. Prove proposition 6.4 from the script.

(5 points)

Exam submission infos: There will be 12 sheets in total. For exam submission you will need a total of 152 points, as 304 will be the total number of points from programming and theory exercise sheets. If you are unsure about how many points you have at the moment+, you can write me a mail (`heinz@dm.uni-bonn.de`) .

Deadline: July 8$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.