Prof. Dr. Stephan Held
Paula Heinz, M. Sc.

Exercise Set 10

Exercise 10.1. Given a directed acyclic graph G (i.e. G might contain undirected cycles) and nonnegative edge weights, show how to compute a maximum weighted set $C \subset E(G)$ such that there is no directed path in G that contains two edges from C, using a maximum flow algorithm. Such a set C is also called an antichain.

(Given a feasible solution for an instance of the Discrete Time-Cost Tradeoff problem, it possibly can be made cheaper along antichains).

(5 points)

Exercise 10.2. Prove proposition 6.1 from the script.

(5 points)

Exercise 10.3. Let $G = (V, E)$ be an undirected graph with non-negative edge weights $w : E \to \mathbb{R}_{\geq 0}$, a set of sinks $T \subset V$, and a root vertex $r \in V \setminus T$. Additionally, we are given required arrival times $rat : T \to \mathbb{R}$. The goal of the Delay Bounded Steiner Tree Problem is to compute a Steiner tree S of $\{r\} \cup T$ in G with minimum weight, such that for each $t \in T$ the length of the unique r-t path in S is at most $rat(t)$. Assuming P $\neq$ NP, show that there is no better than $O(\log(|T|))$-approximation algorithm for this problem.

Hint: You may use that it is NP hard to find an $(1-o(1))\log(n)$-approximation for Set Cover with n elements.

(5 points)

Exercise 10.4. Let $t_1, ..., t_n \in \mathbb{R}^2_{>0}$, $r \coloneqq (0, 0) \in \mathbb{R}^2$, $d(x, y) \coloneqq ||x - y||_1$ and n even.

(a) Show that there exists a perfect matching on $t_1, ..., t_n$ with length at most that of a Steiner arborescence on $t_1, ..., t_n$ rooted in r.

(b) Describe a polynomial time algorithm that computes an $\mathcal{O}(log(n))$-approximation for a minimum length Steiner arborescence on $t_1, ..., t_n$ rooted in r, such that the length of each r-t_i path is $||r - t_i||_1$ $(i = 1, ..., n)$.

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(2+3 points)

Deadline: July 1st, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.