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Exercise Set 9

Exercise 9.1. Prove that the 1D discrete cosine transform linearly reduces to the discrete Fourier transformation. (This shows that it can be solved in $\mathcal{O}(N \log N)$ time.)

(5 points)

Exercise 9.2. Consider the following variant of the SINGLE ROW PLACEMENT WITH FIXED ORDERING problem, in which we minimize the bounding box net length:

Input: A set $\mathcal{C} = \{C_1, \ldots, C_n\}$ of circuits, widths $w(C_i) \in \mathbb{R}_+$, an interval $[0, w(\square)]$, s.t. $\sum_{i=1}^{n} w(C_i) \leq w(\square)$. A netlist $(\mathcal{C}, P, \gamma, \mathcal{N})$ where the offset of a pin $p \in P$ satisfies $x(p) \in [0, w(\gamma(p))]$. Weights $\alpha : \mathcal{N} \to \mathbb{R}_+$.

Task: Find a feasible placement given by a function $x : \mathcal{C} \to \mathbb{R}$ s.t. $0 \leq x(C_1)$, $x(C_i) + w(C_i) \leq x(C_{i+1})$ for $i = 1, \ldots, n-1$ and $x(C_n) + w(C_n) \leq w(\square)$, that minimizes

$$\sum_{N \in \mathcal{N}} \alpha(N) \cdot \mathrm{BB}(N).$$

Here, $\mathrm{BB}(N)$ denotes the bounding box net length.

Show that there exist $f_i : [0, w(\square)] \to \mathbb{R}$, $i = 1, \ldots, n$, piecewise linear, continuous and convex, such that we can solve this problem by means of the SINGLE ROW ALGORITHM.

(5 points)

Exercise 9.3. Consider the PLACEMENT LEGALIZATION PROBLEM with $y_{\max} - y_{\min} = 1$. We are given an infeasible placement $\tilde{x} : \mathcal{C} \to \mathbb{R}$. Show that there are feasible instances for which there is no optimum solution which is consistent with $\tilde{x}$, i.e. such that $x(C) < x(C') \Rightarrow \tilde{x}(C) \leq \tilde{x}(C')$.

(5 points)

Exercise 9.4. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ $(1 \leq i \leq n)$ depicted in Figure 9.1. Assume that the delay θ_i

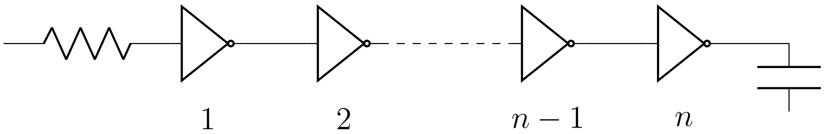

Figure 9.1: Chain of inverters.

through inverter i is given by

$$\theta_i(x) = \alpha + \frac{\beta \cdot x_{i+1}}{x_i} \qquad \text{for } 1 \leq i < n-1$$

where $x = (x_1, \ldots, x_n)$, $\alpha \geq 0$, $\beta > 0$. Wire delays, slews and transitions are ignored.

Derive a closed formula for the size x_i of the i-th inverter in a solution x of the total delay minimization problem for fixed x_1, x_n:

$$\min \left\{ \sum_{i=1}^{n-1} \theta_i(x) : x_i > 0 \text{ for all } 2 \leq i \leq n-1 \right\}.$$

(5 points)

Deadline: June 24th, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.