Exercise Set 8

Exercise 8.1. Consider the 2D inverse discrete cosine transform that for $\hat{f} \in \mathbb{R}^{M\times N}$ computes $f \in \mathbb{R}^{M\times N}$ as

$$f_{i,j} = \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} \alpha_{M,l}\alpha_{N,k}\hat{f}_{l,k} \cos\left[\frac{l\pi}{N}(i+\frac{1}{2})\right] \cos\left[\frac{k\pi}{N}(j+\frac{1}{2})\right]$$

$(i \in \{0,\ldots,M\}, j \in \{0,\ldots,N\})$.

(a) Determine the matrix A_{MN} representing the 2D Poisson discretization with Neumann boundary conditions and the matrix C_{MN} representing the inverse discrete cosine transform $f = C_{MN}\hat{f}$. Use row indices (i,j) and column indices (l,k), $i,l \in \{0,\ldots,M-1\}$ and $j,k \in \{0,\ldots,N-1\}$.

(b) Show that the columns of C_{MN} are eigenvectors of A_{MN} and determine their eigenvalues.

(c) Show that the columns of C_{MN} are orthogonal.

(3+3+3 points)

Exercise 8.2. Show how the electric field vector $\xi = -\nabla\psi$ (from the section on flat non-linear placement) can be computed directly from $\hat{\rho}$, without using u or the finite difference method.

(5 points)

Exercise 8.3. Consider the following wirelength model for a $(\mathcal{C}, P, \gamma, \mathcal{N})$. For a net $N \in \mathcal{N}$,

$$\begin{aligned}\text{SmoothBB}(N) \quad := \quad & \ln\left(\sum_{p\in N} \exp\left(x(\gamma(p)) + x(p)\right)\right) + \ln\left(\sum_{p\in N} \exp\left(-x(\gamma(p)) - x(p)\right)\right) \\ & + \ln\left(\sum_{p\in N} \exp\left(y(\gamma(p)) + y(p)\right)\right) + \ln\left(\sum_{p\in N} \exp\left(-y(\gamma(p)) - y(p)\right)\right).\end{aligned}$$

Prove:

(a) $\text{BB}(N) \leq \text{SmoothBB}(N) \leq \text{BB}(N) + 4\ln|N|$.

(b) $\text{SmoothBB}(N)$ is a convex function in $(x_C)_{C\in\mathcal{C}}$.

(Hint: it is worth simplifying the notation before proving the core mathematical property.)

(3+3 points)

Deadline: June 17$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.