Exercise Set 7

Exercise 7.1. Prove that unless P = NP, there is no polynomial time n^α approximation algorithm for the QUADRATIC ASSIGNMENT PROBLEM for any $\alpha < 1$ even if $w \equiv 1$, $c \equiv 0$, $d : U \times U \to \{0,1\}$ is metric and G is a tree.

Hint: Transformation of 4-Partition, where G is chosen as a collection of stars (one for each item) whose centers are connected to (an additional) common root vertex. U can be chosen as $|U| = |V(G)|$.

(5 points)

Exercise 7.2. Consider quadratic netlength minimization in x-dimension based on the (quadratic) CLIQUE netmodel i.e.

$$\text{CLIQUESQ}(N) := \sum_{\{p,q\}\subseteq N} \frac{w(N)}{|N|-1}\Big(x(p) + x(\gamma(p)) - x(q) - x(\gamma(q))\Big)^2$$

(a) Show that CLIQUESQ can be replaced equivalently by the quadratic STARSQ netmodel

$$\text{STARSQ}(N) := w'(N) \cdot \min\left\{\sum_{p\in N} \left(x(p) + x(\gamma(p)) - c\right)^2 |c \in \mathbb{R}\right\}$$

for an appropriate weight function w'.

(b) For a fixed placement x and a single net N let $l, r \in N$ be defined as $l := \arg\min\{x(p)+x(\gamma(p)) \mid p \in N\}$ and $r := \arg\max\{x(p)+x(\gamma(p)) \mid p \in N\}$. We further define for $p, q \in N$

$$w^{\text{B2B}}_{pq} := \begin{cases} 0 & \text{if } \{p,q\} \cap \{l,r\} = \emptyset, \\ \Big|x(q) + x(\gamma(q)) - x(p) - x(\gamma(p))\Big|^{-1} & \text{else.} \end{cases}$$

Show that the CLIQUESQ netlength with weights w^{B2B} equals the (linear) bounding box netlength for placement x.

(3 + 3 points)

Exercise 7.3. Consider the sequence $(a_i)_{i\in\mathbb{N}}$ as defined in Nesterov's Algorithm: $a_0 := 0$ and $a_{k+1} := \frac{1+\sqrt{4a_k^2+1}}{2}$ for $k \in \mathbb{N}$. Show the following properties:

(a) $a_{k-1}^2 = (a_k - 1)a_k$ and $\frac{k}{2} \leq a_{k-1} \leq k-1$ for all $k \in \mathbb{N}_{>1}$.

(b) The ratio $\frac{a_k - 1}{a_{k+1}}$ increases monotonically and converges to 1.

(2+2 points)

Exercise 7.4. Let $G = (V, E)$ be an undirected graph with edge weights $w : E \to \mathbb{R}_{\geq 0}$ and $k \in \mathbb{N}$. Let $C \subseteq V$ and $f : V \setminus C \to \{1, \dots, k\}$ be a placement function. We are looking for positions $f : C \to \{1, \dots, k\}$ s.t.

$$\sum_{e=\{v,w\}\in E} w(e) \cdot |f(v) - f(w)|$$

is minimum. Note that f is not required to be injective.

Prove that this problem can be solved optimally by solving $k-1$ minimum weight s-t-cut problems in digraphs with $\mathcal{O}(|V|)$ vertices and $\mathcal{O}(|E|)$ edges.

Hint: Consider digraphs $G_j = (V_j, E_j)$ with $V_j := \{s,t\} \cup C$ and

$$\begin{aligned} E_j := &\Big\{(s,v) : \exists\, w \in V \setminus C, f(w) \leq j, \{v,w\} \in E\Big\} \cup \\ &\Big\{(v,w) : v,w \in C, \{v,w\} \in E\Big\} \cup \\ &\Big\{(v,t) : \exists\, w \in V \setminus C, f(w) > j, \{v,w\} \in E\Big\} \end{aligned}$$

(5 points)

Deadline: June 3rd, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.