Prof. Dr. Stephan Held
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Exercise Set 4

Exercise 4.1. Formulate the SIMPLE GLOBAL ROUTING PROBLEM as an integer linear program with a polynomial number of variables and constraints.

(5 points)

Exercise 4.2. Prove that the number of oracle calls after $t \in \mathbb{N}$ phases of the core Resource Sharing Algorithm is bounded by

$$t|\mathcal{C}| + \frac{|\mathcal{R}|}{\epsilon} \ln \frac{||y^{(t)}||_1}{|\mathcal{R}|}.$$

Hint: Proceed similarly to the proof of Lemma 5.11 in the lecture notes.

(5 points)

Exercise 4.3. (a) Find, for every $m \in \mathbb{N}$, an instance of the Min-Max Resource Sharing Problem with m resources ($m = |\mathcal{R}|$) and $\lambda^* > 0$ such that

$$\inf \left\{ \max_{r \in \mathcal{R}} \sum_{C \in \mathcal{N}} (b_C)_r : b_C \in B_C \right\} \geq |\mathcal{R}|\lambda^*.$$

(b) Prove that for every instance of the Min-Max Resource Sharing Problem it holds

$$\inf \left\{ \max_{r \in \mathcal{R}} \sum_{C \in \mathcal{N}} (b_C)_r : b_C \in B_C \right\} \leq |\mathcal{R}|\lambda^*.$$

(2+3 points)

Exercise 4.4. Let $G = (A \dot\cup B, E)$ be a bipartite graph. Assume that there is a matching covering A. Let $\varepsilon > 0$. Use the Resource Sharing Algorithm to find variables $(x_e)_{e \in E} \in [0,1]^{E(G)}$ that satisfy

$$\sum_{e \in \delta(v)} x_e = 1 \qquad \forall v \in A, \qquad \sum_{e \in \delta(w)} x_e \leq 1 + \varepsilon \qquad \forall w \in B$$

within a running time of $\mathcal{O}(|E| \frac{\ln |B|}{\varepsilon^2})$.

(5 points)

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Deadline: May 13$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.