Exercise Set 3

Exercise 3.1. Let T be an instance of the Rectilinear Steiner Tree Problem and $r \in T$. For a rectilinear Steiner tree Y we denote by $f(Y)$ the maximum length of a path from r to any element of $T \setminus \{r\}$ in Y.

(a) Find an instance where no Steiner tree minimizes both length and f.

(b) Consider the problem of finding a shortest Steiner tree Y minimizing $f(Y)$ among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid?

(1 + 4 points)

Exercise 3.2. Given $u, v \in \mathbb{R}^2$, let

$$\mathcal{L}(u, v) := \{x \in \mathbb{R}^2 : \max\{||x - u||_1, ||x - v||_1\} < ||v - u||_1\}.$$

Let Y be a canonical tree that is full. Prove that any pair of distinct edges $(u, v), (x, y) \in E(Y)$ satisfies $\mathcal{L}(u, v) \cap \mathcal{L}(x, y) = \emptyset$.

Remark: This criterion can be used to prune prospective full components.

(5 points)

Exercise 3.3. Let $T \subset \mathbb{R}^2$ be a finite set of terminals, and $S_1, \ldots, S_m \subseteq \mathbb{R}^2$ be rectangular, axis-parallel blockages. Let $S := \bigcup_i S_i$, $\mathring{S}$ denote the interior of S, and let $0 < L \in \mathbb{R}$ be a constant.

A rectilinear Steiner tree Y for T is *reach-aware* if every connected component of $E(Y) \cap \mathring{S}$ has length at most L.

(a) We define the *Hanan grid induced by* $(T, S_1, \ldots, S_m)$ as the usual Hanan grid for $T \cup \{l_i, u_i \mid 1 \leq i \leq m\}$ where l_i (resp. u_i) is the lower left (resp. upper right) corner of S_i.

Prove or disprove: If there is a reach-aware Steiner tree there is always a shortest reach-aware Steiner tree for T that is a subgraph of the Hanan grid induced by $(T, S_1, \ldots, S_m)$.

(b) Prove that it is $\mathcal{NP}$-hard to compute a reach-aware Steiner tree for T that has at most twice the length of an optimum solution, even with the assumption that all terminals $t \in T$ have distance at most L to unblocked area.

Hint: If there are no terminals on blockages this is not $\mathcal{NP}$-hard.

(5 points)

Exercise 3.4. Provide an instance of the Simple Global Routing Problem which admits a fractional solution, but no feasible integral solution. Your instance has to satisfy $w(N, e) \leq u(e)$ for each net N and edge e.

(5 points)

Deadline: May 6th, before the lecture. The websites for lecture and exercises can be found at:

`https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html`

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.