

Exercise Set 2

Exercise 2.1. For a finite set $\emptyset \neq T \subsetneq \mathbb{R}^2$ we define

$$\text{BB}(T) := \max_{(x,y) \in T} x - \min_{(x,y) \in T} x + \max_{(x,y) \in T} y - \min_{(x,y) \in T} y.$$

We denote by $\text{STEINER}(T)$ the length of a shortest rectilinear (i.e. edge lengths acc. to ℓ_1) Steiner tree for T . Moreover let $\text{MST}(T)$ be the length of a minimum spanning tree in the complete graph on T with edge costs ℓ_1 .

Prove that:

- (a) $\text{BB}(T) \leq \text{STEINER}(T) \leq \text{MST}(T)$;
- (b) $\text{STEINER}(T) \leq \frac{3}{2} \text{BB}(T)$ for $|T| \leq 5$;
- (c) There is no $\alpha \in \mathbb{R}$ s.t. $\text{STEINER}(T) \leq \alpha \text{BB}(T)$ for all finite $\emptyset \neq T \subset \mathbb{R}^2$.

(2 + 3 + 2 points)

Exercise 2.2. Let Y be a Steiner tree for terminal set T with $|T| \geq 2$ in which all leaves are terminals. Prove

$$\sum_{t \in T} (|\delta_Y(t)| - 1) = k - 1$$

where k is the number of full components of Y .

(5 points)

Exercise 2.3. Let (G, c, T) be an instance of the STEINER TREE PROBLEM, G connected, $t \in T$ a terminal and $k \in \mathbb{N}$ with $k \geq 1$. For each of the following functions $V(G) \times 2^T \rightarrow \mathbb{R}_{\geq 0}$ decide whether it defines a feasible lower bound for instances of the RECTILINEAR STEINER TREE PROBLEM and prove your statement.

- (a) For two feasible lower bounds lb_a and lb_b , define $\max(\text{lb}_a, \text{lb}_b)$ by

$$\max(\text{lb}_a, \text{lb}_b)(v, I) := \max(\text{lb}_a(v, I), \text{lb}_b(v, I)).$$

- (b) Define $\text{lb}_{\text{BB}}(v, I) := \text{BB}(\{v\} \cup I)$.

- (c) Define $\text{lb}_{\text{mst}}(v, I) := \frac{\text{mst}(\{v\} \cup I)}{2}$. Here $\text{mst}(\{v\} \cup I)$ denotes the cost of a minimal spanning tree in $(G'[\{v\} \cup I], c')$, where (G', c') is the metric closure of (G, c) .

- (d) Define $\text{lb}_k(v, I) := \max \left\{ \text{smt}(J) \mid t \in J \subseteq I \cup \{v\}, |J| \leq k+1 \right\}$ if $t \in I$ and $\text{lb}_k(v, I) := 0$ otherwise.

(2 + 2 + 2 + 2 points)

Deadline: April 24rd, before the exercise class. The websites for lecture and exercises can be found at:

https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.

Noteworthy Fachschaft event:

Want to put your integration skills to the test on May 16, 2025? Then get ready for this year's Bonn Integration Bee! Juicy prizes await!

More info at: www.bibee.rocks

Not in the mood to compete but still into integrals? Feel free to join as a spectator! There will be a small cold drinks stand hosted by the student council.

We're looking forward to seeing you!

P.S.: This time, the integrals are actually solvable! P.P.S.: You can find last year's integrals – including solutions – on our website!