

Exercise Set 1

Exercise 1.1. Consider the placement instance in Figure 1.1. The circuits c_0, \dots, c_5 must be placed pairwise disjointly, in one of the three circuit rows and within the chip area (magenta outline). The grid width is one. Pins are marked by quadratic boxes within the circuits. Nets are described by Steiner trees connecting their pins; Steiner points are drawn as filled circles.

Note that the pin locations are fixed relative to their containing circuit. Mirroring, flipping, and rotating of circuits is not allowed.

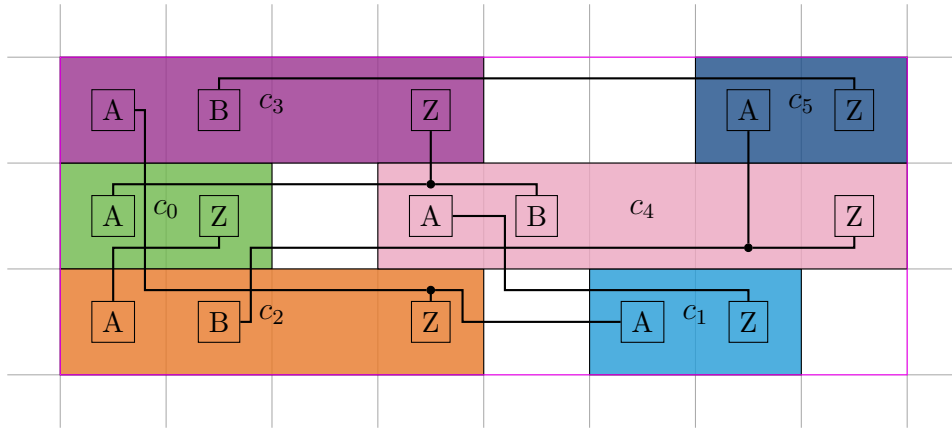


Figure 1.1: A placement instance with suboptimal input placement.

The *Steiner tree netlength* of a placement is defined as

$$\text{steiner}(\mathcal{N}) := \sum_{N \in \mathcal{N}} \text{steiner}(N),$$

where $\text{steiner}(N)$ is the length of a minimum rectilinear Steiner tree connecting all middle points of pin shapes.

Note that for better readability the Steiner trees drawn in Figure 1.1 are *not* minimum. The placement in the figure has a Steiner tree netlength of 32.

The *bounding box netlength* of a placement is defined as

$$\text{BB}(\mathcal{N}) := \sum_{N \in \mathcal{N}} \text{BB}(N),$$

where $\text{BB}(N)$ is half the perimeter of a minimum axis-parallel rectangle that contains all middle points of pin shapes in N .

- (a) Determine the maximum possible absolute difference $|\text{steiner}(\mathcal{N}) - \text{BB}(\mathcal{N})|$ for the given instance and all its legal placements.
- (b) Prove that there is no legal placement with $\text{steiner}(\mathcal{N}) < 9$. Can you find better lower bounds?
- (c) Determine a legal placement of minimum Steiner tree length. (The number of awarded points is reduced by the deviation from the optimum solution.)

(2 + 2 + 6 points)

Exercise 1.2. Let $n \in \mathbb{N}$, $n \geq 7$. Let B_n be the set of *Boolean functions* $f : \{0, 1\}^n \rightarrow \{0, 1\}$ and $A_n := \left\{ f \in B_n \mid \exists \text{ netlist realizing } f \text{ with at most } \frac{2^{n-1}}{n} \text{ circuits of at most two inputs} \right\}$. Prove that

$$\frac{|A_n|}{|B_n|} < \frac{1}{n^{\frac{2^{n-1}}{n}}}.$$

(5 points)

Exercise 1.3. Prove or disprove that for every netlist with technology mapping there is a logically equivalent one that only contains

- (a) *NORs*
- (b) *XORs*
- (c) *NANDs*.

(5 points)

Deadline: April 17th, before the lecture. The websites for lecture and exercises can be found at:

https://www.or.uni-bonn.de/lectures/ss25/chipss25_ex.html

In case of any questions feel free to contact me at heinz@dm.uni-bonn.de.