Exercise Set 12

Exercise 12.1. Let $\alpha > 1$ and $1 \leq \beta < 1+2/(\alpha-1)$. Construct a connected, planar graph G with $w : E(G) \to \mathbb{R}_+$ and $r \in V(G)$ that contains no spanning tree T with the following properties:

(a) For each $v \in V(G)$: $\text{dist}_{w,T}(r,v) \leq \alpha \cdot \text{dist}_{w,G}(r,v)$.
(b) For a minimum-spanning tree M: $\sum_{e \in E(T)} w(e) \leq \beta \cdot \sum_{e \in E(M)} w(e)$.

(5 points)

Exercise 12.2. Given a root $r \in \mathbb{R}^2$, a finite set of sinks $S \subset \mathbb{R}^2$, Lagrangean multipliers $(\lambda_s)_{s \in S}$, the rectilinear cost-distance Steiner arborescence problem asks for a Steiner arborescence Y rooted at r, minimizing

$$\sum_{(v,w) \in E(Y)} ||v-w||_1 + \sum_{s \in S} \lambda_s \cdot \left(\sum_{(v,w) \in E(Y_{[r,s]})} ||v-w||_1 \right)$$

Using the light-approximate shortest path tree algorithm, approximate this problem up to a factor of 3 in $\mathcal{O}(n \log n)$.

(5 points)

Exercise 12.3. Let $G = (V, E)$ be an undirected graph with non-negative weights $w : V \to \mathbb{R}$, a set of sinks $T \subset V$, and a root vertex $r \in V \setminus T$. Additionaly, we are given required arrival times $rat : T \to \mathbb{R}$. The goal of the Delay Bounded Steiner Tree Problem is to compute a Steiner tree S of $\{r\} \cup T$ in G with minimum weight, such that for each $t \in T$ the length of the unique r-t path in S is at most $rat(t)$. Assuming P $\neq$ NP, show that there is no $\Omega(\log(|T|))$-approximation algorithm for this problem.

Hint: You may use that it is NP hard to find an $\Omega(\log(n))$-approximation for Set Cover with n sets.

(5 points)

Exercise 12.4. Let $t_1, ..., t_n \in \mathbb{R}^2_{>0}$, $r := (0,0) \in \mathbb{R}^2$, $d(x,y) := ||x-y||_1$.

(a) Show that there exists a perfect matching on $t_1, ..., t_n$ with length at most that of a Steiner arborescence on $t_1, ..., t_n$ rooted in r.

(b) Describe a polynomial time algorithm that computes an $\mathcal{O}(log(n))$-approximation for a minimum length Steiner arborescence on $t_1, ..., t_n$ rooted in r, such that the length of each r-t_i path is $||r - t_i||_1$ ($i = 1, ..., n$).

(2+3 points)

Deadline: July 9th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.