Exercise Set 11

Exercise 11.1. Show that the VERTEX-DISJOINT PATHS PROBLEM is NP-complete even if G is a subgraph of a track graph G_T with two routing planes. Recall that in this case G_T is a graph $G_T = (V, E)$ for some $n_x, n_y \in \mathbb{N}$ with $V = \{1, \dots, n_x\} \times \{1, \dots, n_y\} \times \{1, 2\}$ and $E = \{\{(x, y, z), (x', y', z')\} : |x - x'|z + |y - y'|(3 - z) + |z - z'| = 1\}$.

Hint: Consider the proof of Theorem 5.2.

(5 points)

Exercise 11.2. Consider the ESCAPE ROUTING PROBLEM: We are given a complete 2-dimensional grid graph $G = (V, E)$ (i.e. $V = \{0, \dots, k-1\} \times \{0, \dots, k-1\}$ and $E = \{\{v, w\} \mid v, w \in V, \|v - w\| = 1\})$ and a set $P = \{p_1, \dots, p_m\} \subseteq V$. The task is to compute vertex-disjoint paths $\{q_1, \dots, q_m\}$ s.t. each q_i connects p_i with a point on the border $B = \{(x, y) \in V \mid \{x, y\} \cap \{0, k-1\} \neq \emptyset\}$.

Find a polynomial-time algorithm for the ESCAPE ROUTING PROBLEM or prove that the problem is NP-hard.

(5 points)

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Exercise 11.3. In this exercise, we use the setting of section 5.5.2 in the lecture notes. Let $c\colon E \to \mathbb{R}_{>0}$ depend only on direction and layer.

(a) Let $s = (x_s, y_s, z_s) \in V$, $y_v \in \mathbb{Z}$ and $v_i = (x_s, y_v, i) \in V$ for $i = 1, \ldots, l$. Show that shortest paths from s to v_i for $i = 1, \ldots, l$ can be computed in $\mathcal{O}(l)$ total time (so not just individually in linear time, but all l).

(b) Show that, without preprocessing time, one can compute $\mathrm{dist}_{G,c}(s, T)$ for any given $s \in V$ and given $T \subseteq V$ consisting of t rectangles in $\mathcal{O}(tl)$ time (you may use the first part as a black box).

(5+5 points)

Deadline: July 2^{nd}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.