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Exercise Set 10

Exercise 10.1. Provide an instance of the SIMPLE GLOBAL ROUTING PROBLEM which admits a fractional solution, but no feasible integral solution. Your instance has to satisfy $w(N, e) \leq u(e)$ for each net N and edge e.

(5 points)

Exercise 10.2. Formulate the SIMPLE GLOBAL ROUTING PROBLEM as an integer linear program with a polynomial number of variables and constraints.

(5 points)

Exercise 10.3. Let $G = (A \dot{\cup} B, E)$ be a bipartite graph. Assume that there is a matching covering A. Let $\varepsilon > 0$. Use the Resource Sharing Algorithm to find variables $(x_e)_{e \in E} \in [0,1]^{E(G)}$ that satisfy

$$\sum_{e \in \delta(v)} x_e = 1 \qquad \forall v \in A, \qquad \sum_{e \in \delta(w)} x_e \leq 1 + \varepsilon \qquad \forall w \in B$$

within a running time of $\mathcal{O}(|E| \frac{\ln |B|}{\varepsilon^2})$.

(5 points)

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Exercise 10.4. Given an instance of the Min-Max Resource Sharing Problem with σ-optimal block solvers for some fixed $\sigma \geq 1$.

(a) Show that t phases of the Resource Sharing Algorithm call the oracle at most

$$\min\left\{t\Lambda,\ t|\mathcal{C}| + \frac{|\mathcal{R}'|}{\varepsilon}\ln\left(\mathbb{1}^\top y^{(t)}\right)\right\}$$

times where $\Lambda := \sum_{C\in\mathcal{C}} \max\{1, \sup\{b_r \mid r \in \mathcal{R}, b \in \mathcal{B}_C\}\}$ and $\mathcal{R}' := \{r \in \mathcal{R} \mid \exists C \in \mathcal{C}, b \in \mathcal{B}_C \text{ with } b_r > 1\}$.

(b) Prove that a $\sigma(1+\omega)$-approximate solution can be computed in

$$O\Big(\theta \log|\mathcal{R}| \left(\left(|\mathcal{C}| + |\mathcal{R}|\right)\log\log|\mathcal{R}| + \omega^{-2}\min\left\{\rho|\mathcal{C}|, |\mathcal{C}| + |\overline{\mathcal{R}}|\right\}\right)\Big)$$

time where $\rho := \max\{1, \sup\{b_r/\lambda^* \mid r \in \mathcal{R}, C \in \mathcal{C}, b \in \mathcal{B}_C\}\}$ and $\overline{\mathcal{R}} := \{r \in \mathcal{R} \mid \exists C \in \mathcal{C}, b \in \mathcal{B}_C \text{ with } b_r > \lambda^*\}$.

Remark: For practical routing instances ρ and $|\overline{\mathcal{R}}|$ are usually small.

(2 + 4 points)

Deadline: June 25$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.