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Exercise Set 9

Exercise 9.1. Prove that the 1D discrete cosine transform linearly reduces to the discrete Fourier transformation (This shows that it can be solved in $\mathcal{O}(N \log N)$ time). See also in the lecture notes.

(5 points)

Exercise 9.2. Consider the Placement Legalization Problem with $y_{\max} - y_{\min} = 1$. We are given an infeasible placement $\tilde{x} : \mathcal{C} \to \mathbb{R}$. Show that there are feasible instances for which there is no optimum solution which is consistent with $\tilde{x}$, i.e. such that $x(C) < x(C') \Rightarrow \tilde{x}(C) \leq \tilde{x}(C')$.

(5 points)

Exercise 9.3. Consider the following variant of the Single Row Placement With Fixed Ordering problem, in which we minimize the bounding box net length:

> **Input:** A set $\mathcal{C} = \{C_1, \ldots, C_n\}$ of circuits, widths $w(C_i) \in \mathbb{R}_+$, an interval $[0, w(\Box)]$, s.t. $\sum_{i=1}^{n} w(C_i) \leq w(\Box)$. A netlist $(\mathcal{C}, P, \gamma, \mathcal{N})$ where the offset of a pin $p \in P$ satisfies $x(p) \in [0, w(\gamma(p))]$. Weights $\alpha : \mathcal{N} \to \mathbb{R}_+$.
>
> **Task:** Find a feasible placement given by a function $x : \mathcal{C} \to \mathbb{R}$ s.t. $0 \leq x(C_1)$, $x(C_i) + w(C_i) \leq x(C_{i+1})$ for $i = 1, \ldots, n-1$ and $x(C_n) + w(C_n) \leq w(\Box)$, that minimizes
>
> $$\sum_{N \in \mathcal{N}} \alpha(N) \cdot \mathrm{BB}(N).$$

Here, $\mathrm{BB}(N)$ denotes the bounding box net length.

Show that there exist $f_i : [0, w(\Box)] \to \mathbb{R}$, $i = 1, \ldots, n$, piecewise linear, continuous and convex, such that we can solve this problem by means of the Single Row Algorithm.

(5 points)

Exercise 9.4. Consider the following algorithm to compute a rectilinear Steiner tree Y for a set T of points in the plane $\mathbb{R}^2$.

Choose $t \in T$ arbitrarily;
$Y := (\{t\}, \emptyset), S := T \setminus \{t\}$
while $S \neq \emptyset$ **do**
  Choose $s \in S$ with minimum $dist(s, Y)$
  if $E(Y) = \emptyset$ **then**
    $Y := (\{t, s\}, \{\{t, s\}\})$
  else
    Let $\{u, w\} \in E(Y)$ be an edge which minimizes $dist(s, SP(u, w))$
    $v := \arg\min\{dist(s, v) \mid v \in SP(u, w)\}$
    $Y := (V(Y) \cup \{v\} \cup \{s\}, E(Y) \setminus \{\{u, w\}\} \cup \{\{u, v\}, \{v, w\}, \{v, s\}\})$
  end if
  $S := S \setminus \{s\}$
end while

In this notation $SP(u, w) \subset \mathbb{R}^2$ is the area covered by shortest paths between u and w, and $dist(s, Y)$ is the minimum distance between s and the shortest path area $SP(u, w)$ of an edge $\{u, w\} \in E(Y)$.

Show that the algorithm is a $\frac{3}{2}$-approximation algorithm for the MINIMUM STEINER TREE PROBLEM.

(5 points)

Deadline: June 18th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.