Exercise Set 8

Exercise 8.1. Consider the fractional MULTISECTION PROBLEM with $m = 2$ regions and n circuits. Provide an alternative, simple (not using network flows) $\mathcal{O}(n \log n)$ algorithm that computes an optimum fractional partition with the additional property that all but one circuit are assigned to only one region.

(5 points)

Exercise 8.2. The GRIDDED PLACEMENT PROBLEM is an extension of the STANDARD PLACEMENT PROBLEM with a grid $\Gamma = \Gamma_x \times \Gamma_y$ where $\Gamma_z := \{k \cdot \delta_z : k \in \mathbb{Z}\}$ with $\delta_z \in \mathbb{Z}$ for $z \in \{x, y\}$. In this variant, the lower left corner of each circuit is required to be in Γ.

Prove that the GRIDDED PLACEMENT PROBLEM is NP-hard even if an optimum solution of the associated ungridded placement problem is known.

(5 points)

Exercise 8.3. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable and convex function where the gradient is Lipschitz continuous with constant L. Assume that L is known in advance and replace the variable step length by a constant step length $\theta_k = \frac{1}{L}$ ($k \in \{1, \dots, K\}$). Prove that for $\epsilon > 0$ the modified algorithm finds a solution x with $f(x) - f(x^\star) < \epsilon$ using at most $t_g = ||y_0 - x^\star|| \sqrt{2L/\epsilon}$ gradient evaluations and zero function evaluations. (Proving that $||y_0 - x^\star|| \sqrt{4L/\epsilon}$ gradient computations are sufficient yields half the points).

(5 points)

Prof. Dr. Stephan Held
Niklas Schlomberg, M. Sc.

Exercise 8.4. Consider the 2D inverse discrete cosine transform that for $\hat{f} \in \mathbb{R}^{M \times N}$ computes $f \in \mathbb{R}^{M \times N}$ as

$$f_{i,j} = \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} \alpha_{M,l} \alpha_{N,k} \hat{f}_{l,k} \cos\left[\frac{l\pi}{N}(i + \frac{1}{2})\right] \cos\left[\frac{k\pi}{N}(j + \frac{1}{2})\right]$$

$(i \in \{0, \dots, M\}, j \in \{0, \dots, N\})$.

(a) Determine the matrix A_{MN} representing the 2D Poisson discretization with Neumann boundary conditions and the matrix C_{MN} representing the inverse discrete cosine transform $f = C_{MN}\hat{f}$. Use row indices (i, j) and column indices (l, k), $i, l \in \{0, \dots, M-1\}$ and $j, k \in \{0, \dots, N-1\}$.

(b) Show that the columns of C_{MN} are eigenvectors of A_{MN} and determine their eigenvalues.

(c) Show that the columns of C_{MN} are orthogonal.

(3+3+3* points)

Deadline: June 11th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.