Prof. Dr. Stephan Held
Niklas Schlomberg, M. Sc.

Exercise Set 7

Exercise 7.1. Consider quadratic netlength minimization in x-dimension based on the (quadratic) Clique netmodel i.e.

$$\textsc{Clique}\text{SQ}(N) := \sum_{\{p,q\} \subseteq N} \frac{w(N)}{|N|-1} \Big(x(p) + x(\gamma(p)) - x(q) - x(\gamma(q))\Big)^2$$

(a) Show that CliqueSQ can be replaced equivalently by the quadratic StarSQ netmodel

$$\textsc{Star}\text{SQ}(N) := w'(N) \cdot \min \left\{ \sum_{p \in N} (x(p) + x(\gamma(p)) - c)^2 \,|c \in \mathbb{R} \right\}$$

for an appropriate weight function w'.

(b) For a fixed placement x and a single net N let $l, r \in N$ be defined as $l := \arg\min\{x(p) + x(\gamma(p)) \mid p \in N\}$ and $r := \arg\max\{x(p) + x(\gamma(p)) \mid p \in N\}$. We further define for $p, q \in N$

$$w_{pq}^{\text{B2B}} := \begin{cases} 0 & \text{if } \{p,q\} \cap \{l,r\} = \emptyset, \\ \Big|x(q) + x(\gamma(q)) - x(p) - x(\gamma(p))\Big|^{-1} & \text{else.} \end{cases}$$

Show that the CliqueSQ netlength with weights w^{B2B} equals the (linear) bounding box netlength for placement x.

(3 + 3 points)

Exercise 7.2. The Minimum Cut Linear Arrangement Problem is defined as follows: Given a hypergraph $G = (V, E)$ where $E \subseteq \mathcal{P}(V)$, find a bijective mapping $f : V \to \{1, ..., |V|\}$ that minimizes

$$\max_{i \in \{1,...,|V|-1\}} \Big|\Big\{e \in E : \exists v, w \in e \text{ s.t. } f(v) \leq i < f(w)\Big\}\Big|$$

Show that this problem can be solved in $O(nm2^n)$ where $n|V|, m|E|$.

(5 points)

Exercise 7.3. Consider the following wirelength model for a $(\mathcal{C}, P, \gamma, \mathcal{N})$. For a net $N \in \mathcal{N}$,

$$\begin{aligned}\text{SmoothBB}(N) \quad := \quad & \ln\left(\sum_{p\in N} \exp\left(x(\gamma(p)) + x(p)\right)\right) + \ln\left(\sum_{p\in N} \exp\left(-x(\gamma(p)) - x(p)\right)\right) \\ & + \ln\left(\sum_{p\in N} \exp\left(y(\gamma(p)) + y(p)\right)\right) + \ln\left(\sum_{p\in N} \exp\left(-y(\gamma(p)) - y(p)\right)\right).\end{aligned}$$

Prove:

(a) $\text{BB}(N) \leq \text{SmoothBB}(N) \leq \text{BB}(N) + 4\ln|N|$.

(b) $\text{SmoothBB}(N)$ is a convex function in $(x_C)_{C\in\mathcal{C}}$.

(Hint: it is worth simplifying the notation before proving the core mathematical property.)

(2+3 points)

Exercise 7.4. Consider the sequence $(a_i)_{i\in\mathbb{N}}$ as defined in Nesterov's Algorithm: $a_0 := 0$ and $a_{k+1} := \frac{1+\sqrt{4a_k^2+1}}{2}$ for $k \in \mathbb{N}$. Prove the following two properties:

(a) $a_{k-1}^2 = (a_k - 1)a_k$ and $\frac{k}{2} \leq a_{k-1} \leq k-1$ for all $k \in \mathbb{N}$.

(b) The ratio $\frac{a_k - 1}{a_{k+1}}$ increases monotonically and converges to 1.

(2 + 2 points)

Deadline: June 4^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.