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Exercise Set 6

Exercise 6.1. Let N be a finite set of pins, and let S_p be a set of axis-parallel rectangles for each $p \in N$. We want to compute the *bounding box netlength* of N, i.e. an axis-parallel rectangle R with minimum perimeter s.t. for every $p \in N$ there is an $S \in S_p$ with $R \cap S \neq \emptyset$.

Show how to compute such a rectangle in $O(n^3)$ time where $n := \sum_{p \in N} |S_p|$.

(5 points)

Exercise 6.2. Let T be a finite, nonempty subset of $\mathbb{R}^2$. Show that Clique can be computed in $O(|T| \log |T|)$ time where

$$\textsc{Clique}(T) := \frac{1}{|T| - 1} \sum_{\{(x,y),(x',y')\} \subseteq T} (|x - x'| + |y - y'|) \,.$$

(4 points)

Exercise 6.3. Prove that the Standard Placement Problem can be solved optimally in

$$O\Big(\big((n+s)!\big)^2 \big((m + n^2 + k \log k)(n+k) \log(n+k) + (sn)\big)\Big)$$

time, where $n := |\mathcal{C}|$, $k := |\mathcal{N}|$, $m := |\mathcal{P}|$ and $s := |\mathcal{S}|$.

(6 points)

Exercise 6.4. Let $G = (V, E)$ be an undirected graph with edge weights $w : E \to \mathbb{R}_{\geq 0}$ and $k \in \mathbb{N}$. Let $C \subseteq V$ and $f : V \setminus C \to \{1, \dots, k\}$ be a placement function. We are looking for positions $f : C \to \{1, \dots, k\}$ s.t.

$$\sum_{e=\{v,w\} \in E} w(e) \cdot |f(v) - f(w)|$$

is minimum. Note that f is not required to be injective.

Prove that this problem can be solved optimally by solving $k - 1$ minimum weight s-t-cut problems in digraphs with $\mathcal{O}(|V|)$ vertices and $\mathcal{O}(|E|)$ edges.

Hint: Consider digraphs $G_j = (V_j, E_j)$ with $V_j := \{s, t\} \cup C$ and

$$\begin{aligned} E_j := &\Big\{(s, v) : \exists\ w \in V \setminus C, f(w) \leq j, \{v, w\} \in E\Big\} \cup \\ &\Big\{(v, w) : v, w \in C, \{v, w\} \in E\Big\} \cup \\ &\Big\{(v, t) : \exists\ w \in V \setminus C, f(w) > j, \{v, w\} \in E\Big\} \end{aligned}$$

(5 points)

Deadline: Mai 28th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.