# Exercise Set 2

**Exercise 2.1.** For $f \in B_n$ let $S_{\text{DNF}}(f)$ be the minimum number of literals needed in any DNF representation of $f$. Show that $\max\{S_{\text{DNF}}(f) \mid f \in B_n\} \in \Omega(2^n)$.

(5 points)

**Exercise 2.2.** A Boolean function $f \in B_n$ *depends essentially* on all its variables if for every $1 \leq i \leq n$ the subfunctions $f|_{x_i=0}$ and $f|_{x_i=1}$ are different.

Let $f \in B_n$ be a function that essentially depends on all its variables. Show:

(a) $S_{B_2}(f) \geq n - 1$,

(b) $D_{B_2}(f) \geq \lceil \log_2 n \rceil$.

(5 points)

**Exercise 2.3.** Let $f \in B_n$ be a Boolean function given as an oracle (i.e. for each $x \in \{0,1\}^n$ the value $f(x)$ can be computed in $\mathcal{O}(1)$ time). Show that the set $PI(f)$ of all prime implicants can be computed in $\mathcal{O}(n^2 3^n)$ time.

(5 points)

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**Exercise 2.4.** Assume unit $B_2$-circuit-delay and zero wire-delay.

(a) Show that for $n$ inputs with arrival times $t_i \in \mathbb{N}$ $(i = 1, \dots, n)$ there are $n$-ary AND, OR or XOR circuits over $B_2$ with delay $d \in \mathbb{N}$ if and only if

$$\sum_{i=1}^{n} 2^{t_i - d} \leq 1.$$

(b) Provide an algorithm that finds such a circuit in $\mathcal{O}(n \log n)$ time.

(3 + 3 points)

**Deadline:** April 23rd, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss24/chipss24_ex.html`

In case of any questions feel free to contact me at schlomberg@or.uni-bonn.de.