Research Institute for Discrete Mathematics
Chip Design
Summer Term 2014

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Exercise Set 11

Exercise 11.1:
Consider the LP formulation of the HITCHCOCK TRANSPORTATION PROBLEM

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} \cdot x_{ij} & \\ s.t. \quad & \sum_{j=1}^{m} x_{ij} = a_i & \forall\, i = 1, ..., n, \\ & \sum_{i=1}^{n} x_{ij} = b_j & \forall\, j = 1, ..., m, \\ & x_{ij} \geq 0 & \forall\, i = 1, ..., n,\ j = 1, ..., m, \end{aligned}$$

and its dual

$$\max \left\{ \sum_{i=1}^{n} a_i \cdot u_i + \sum_{j=1}^{m} b_j \cdot v_j : u_i + v_j \leq c_{ij}\ \forall\, (i,j) \in \{1, ..., n\} \times \{1, ..., m\} \right\}.$$

Prove: Given an optimum dual solution (u, v), an optimum primal solution can be computed in $\mathcal{O}(n)$ time if m is considered to be constant.

Exercise 11.2:
Show how the above LP with $m = 2$ can be solved in $\mathcal{O}(n \log n)$ time without calling any min-cost flow algorithm.

Deadline: Thursday, July 3, before the lecture.
The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss14/ss14.html

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de .