Research Institute for Discrete Mathematics
Chip Design
Summer Term 2014

Prof. Dr. S. Hougardy
R. Scheifele, M. Sc.

Exercise Set 10

Exercise 10.1:

Prove: For $d = 2$, the optimum objective function value of the spreading LP is a lower bound for the optimum objective function value of the corresponding instance of the 2-DIMENSIONAL ARRANGEMENT PROBLEM.

(4 points)

Exercise 10.2:

Consider the special case of the QUADRATIC ASSIGNMENT PROBLEM where $|U| = |V(G)|$, $w(e) = 1$ for all $e \in E(G)$, d is metric, c is zero, and G is a *wheel*, i.e. for even n we have $V(G) = \{v_1, ..., v_n\}$ and $E(G) = E_1 \cup E_2$ with $E_1 = \{\{v_i, v_{i+1}\} : i = 1, ..., n\}$ and $E_2 = \{\{v_i, v_{i+\frac{n}{2}}\} : i = 1, ..., \frac{n}{2}\}$, where all indices are modulo n. Let f^* be the embedding such that $\{\{f^*(x), f^*(y)\} : \{x, y\} \in E_1\}$ is a shortest TSP tour on U with respect to d.

a) Show that $\sum_{e=(x,y)\in E(G)} d(f^*(x), f^*(y)) = \Omega(n \cdot OPT)$, where OPT denotes the optimum objective function value of the given instance of the QUADRATIC ASSIGNMENT PROBLEM.

b) Give a polynomial time 3-approximation algorithm for the above special case of the QUADRATIC ASSIGNMENT PROBLEM.

(4 + 4 points)

Exercise 10.3:

Let $G = (V, E)$ be a simple undirected graph with $V = \{1, \ldots, n\}$. The *Laplacian matrix* L_G of G is the $n \times n$-matrix whose entries $l_{i,j}$, $1 \le i, j \le n$, are given by

$$l_{i,j} = \begin{cases} -1 & \text{if } \{i,j\} \in E, \\ |\delta(i)| & \text{if } i = j, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

(a) Prove that L_G is positive semidefinite, that is, $x^T L_G x \ge 0$ for all $x \in \mathbb{R}^n$.

(b) Let G be connected and let $\lambda_1 \le \lambda_2 \le \ldots \le \lambda_n$ be the eigenvalues of L_G. Show that $\lambda_1 = 0$ and $\lambda_2 > 0$.

(c) Show that the multiplicity of 0 as an eigenvalue of L_G equals the number of connected components of G.

(1 + 1 + 2 points)

Deadline: Thursday, June 26, before the lecture.
The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss14/ss14.html

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de .

Note that this will be the last exercise sheet (except for programming exercises) that will be relevant for admittance to the exam.