Research Institute for Discrete Mathematics
Chip Design
Summer Term 2014

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Exercise Set 2

Exercise 2.1:
Prove that the following problem is NP-complete for every constant $\alpha \geq 1$:

Input:	A set $\{[0, w_i] \times [0, h_i] : i = 1, ..., n\}$ of rectangular circuits and a rectangular chip area $[0, w] \times [0, h]$ such that $\alpha \cdot \sum_{i=1}^{n} w_i h_i \leq wh$.
Task:	Decide whether there exists a feasible placement.

(4 points)

Exercise 2.2:
Given a set $\{[x_{i_1}, x_{i_2}] \times [y_{i_1}, y_{i_2}] : i = 1, ..., n\}$ of axis-parallel line segments (i.e. $x_{i_1} = x_{i_2}$ or $y_{i_1} = y_{i_2}$ for all $i = 1, ..., n$), give an algorithm that computes all pairs of intersecting line segments in $\mathcal{O}(n \log(n) + k)$ time, where k is the number of intersecting pairs.

(4 points)

Exercise 2.3:
Consider the Steiner Tree Problem in Graphs:

Input:	A connected undirected graph $G = (V, E)$, weights $c : E \to \mathbb{R}_{\geq 0}$ and a set $T \subseteq V$.
Task:	Find a minimum weight Steiner tree for T in G.

Give a $2\left(1 - \frac{1}{|T|}\right)$ approximation algorithm for the above problem with running time $\mathcal{O}(n \cdot (n \log n + m))$ for $n := |V|$ and $m := |E|$.

(3 points)

Exercise 2.4:
For a finite non-empty set $T \subseteq \mathbb{R}^2$ we define

$$BB(T) := \max_{(x,y)\in T} x - \min_{(x,y)\in T} x + \max_{(x,y)\in T} y - \min_{(x,y)\in T} y$$

$$smt(T) := \text{length of a shortest rectilinear Steiner tree for } T$$

Prove:

a) $smt(T) \leq \frac{3}{2} BB(T)$ for all $T \subseteq \mathbb{R}^2$ with $|T| \leq 5$.

b) There exists no $k \in \mathbb{N}$ with $smt(T) \leq k \cdot BB(T)$ for all finite $T \subseteq \mathbb{R}^2$.

(3 + 2 points)

Deadline: Thursday, April 24, before the lecture.
The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss14/ss14.html

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de .