Research Institute for Discrete Mathematics
Chip Design
Summer term 2013

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Exercise Set 8

Exercise 8.1:
Show that for quadratic netlength minimization CLIQUE net models can be replaced equivalently by STAR net models by adjusting net weights.
Conclude that for quadratic netlength minimization it suffices to solve a linear equation system $Ax = b$, where the number of non-zero entries of A can be bounded by a linear function in the number of pins and circuits.

(4 points)

Exercise 8.2:
Let $G = (V, E)$ be a simple undirected graph with $V = \{1, \dots, n\}$. The *Laplacian matrix* L_G of G is the $n \times n$-matrix whose entries $l_{i,j}, 1 \le i, j \le n$, are given by

$$l_{i,j} = \begin{cases} -1 & \text{if } \{i, j\} \in E, \\ |\delta(i)| & \text{if } i = j, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

(a) Prove that L_G is positive semidefinite, that is, $x^T L_G x \ge 0$ for all $x \in \mathbb{R}^n$.

(b) Let G be connected and let $\lambda_1 \le \lambda_2 \le \ldots \le \lambda_n$ be the eigenvalues of L_G. Show that $\lambda_1 = 0$ and $\lambda_2 > 0$.

(c) Show that the multiplicity of 0 as an eigenvalue of L_G equals the number of connected components of G.

(1+1+2 points)

Exercise 8.3:
Let G be a graph as in Exercise 8.2 and let λ_2 be the second smallest eigenvalue of L_G. For $\emptyset \ne A \subsetneq V(G)$ define the sparsity of A as

$$sp(A) := \frac{|\delta(A)|}{\min\{|A|, |V(G) \setminus A|\}}.$$

Show that $\lambda_2 \le 2 \cdot \min\{sp(A) : \emptyset \ne A \subsetneq V(G)\}$.

(4 points)

Exercise 8.4:
Show that the Single Row Algorithm can be used to minimize the (linear) bounding box netlength instead of the quadratic movement for the Single Row Placement With Fixed Ordering.

(4 points)

Deadline: Thursday, June 27th, before the lecture.