Research Institute for Discrete Mathematics
Chip Design
Summer term 2013

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Exercise Set 5

Exercise 5.1:
For a finite set $V \subseteq \mathbb{R}^2$, the ℓ_1−Voronoi diagram consists of the regions

$$P_v := \left\{ x \in \mathbb{R}^2 \ : \ ||x - v||_1 = \min_{w \in V} ||x - w||_1 \right\}$$

for $v \in V$. The ℓ_1−Delaunay triangulation of V is the graph

$$(V,\ \{\{v, w\} \subseteq V,\ v \neq w,\ |P_v \cap P_w| > 1\}).$$

For simplicity you can assume that the slope of each straight line connecting two elements of V is neither 1 nor -1.

(a) Show that the ℓ_1−Delaunay triangulation is a planar graph.

(b) Show that the ℓ_1−Delaunay contains a minimum-length spanning tree.

(c) Describe a $\frac{3}{2}$-factor approximation algorithm for the Rectilinear Steiner Tree Problem running in $O(|V| \log |V|)$ using the fact that the Delaunay triangulation can be computed in $O(|V| \log |V|)$ time.

(6 points)

Definition:
For $t \in \mathbb{R}^2$, we denote the x- (res. y-) coordinate of t by $x(t)$ (resp. $y(t)$).

Exercise 5.2:
Let $T \subset \mathbb{R}^2$ be a finite set of terminals.

(a) Let $t' \in T$ such that $x(t') < \min\{x(t) : t \in T \backslash \{t'\}\}$. Define $t'' = (\min\{x(t) : t \in T \backslash \{t'\}\}, y(t'))$. Show that there exists a shortest rectilinear Steiner tree for T consisting of a shortest rectilinear Steiner tree for $(T \backslash \{t'\}) \cup \{t''\}$ plus the edge $\{t', t''\}$.

(b) Let H be the Hanan grid of T and let $E' \subseteq E(H[T])$ be a set of edges such that no two vertical edges share the same row and no two horizontal edges share the same column of H. Show that there exists a shortest rectilinear Steiner tree for T containing all edges in E'.

(1+3 points)

Exercise 5.3:
Let $T \subset \mathbb{R}^2$ be a finite set of terminals located on k parallel horizontal lines (i.e. $|\{y(t) : t \in T\}| = k$). We assume that the elements of T are sorted by their x-coordinate in non-decreasing order. Prove:

(a) If $k = 2$, a shortest rectilinear Steiner tree for T can be found in linear time.

(b) If k is constant and on each of the k parallel lines there is a terminal with x-coordinate $\min\{x(t) : t \in T\}$, a shortest rectilinear Steiner tree for T can be found in linear time.

(4+2 points)

Exercise 5.4:
Given a chip area A and a set $\mathcal{C}$ of circuits. A *move bound* for a circuit C is a subset $A_C \subseteq A$ in which C *must* be placed entirely. Assume $h(C) = w(C) = 1$ for all $C \in \mathcal{C}$ and that A and each move bound A_C, $C \in \mathcal{C}$, are axis-parallel rectangles with integral coordinates.
Describe an algorithm with running time polynomial in $|\mathcal{C}|$ that decides whether there exists a feasible placement such that all move bound constraints are met.
(4 points)

Exercise 5.5:
Show that the following decision problem is NP-complete:

Instance: $k \in \mathbb{N}$ and an instance of the STANDARD PLACEMENT PROBLEM without blockages where all circuits have unit width and unit height and all net weights are equal to 1.

Question: Is there a feasible solution with bounding box net length at most k?
(4 points)

Deadline: Thursday, June **6**th, before the lecture.