Research Institute for Discrete Mathematics
Chip Design
Summer term 2013

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Exercise Set 1

Exercise 1.1:
Let $n \in \mathbb{N}$ such that $\log_2(n) \in \mathbb{N}$ and let $+ : \{0,1\}^{2n} \to \{0,1\}^{n+1}$ be the addition function of two binary n-bit integers:

> **Input:** A_i, $B_i \in \{0,1\}$ for $i = 0, 1, \ldots, n-1$ representing $A = \sum_{i=0}^{n-1} 2^i \cdot A_i$ and $B = \sum_{i=0}^{n-1} 2^i \cdot B_i$.
>
> **Output:** The binary representation of $A + B$.

Construct two netlists (one for condition a) and one for condition b)) realizing the function + using a library containing *AND*s, *OR*s and *XOR*s such that

a) The number of used circuits is at most $5n$.

b) The number of circuits on each path from an input pin to an output pin is at most $n + \log_2(n)$.

For both netlists derive formulas for the number of used circuits and the number of circuits on the longest path from an input pin to an output pin.

(6 points)

Exercise 1.2:
Prove or disprove that for every netlist with technology mapping there is a logically equivalent one that only contains a) *NOR*s b) *XOR*s c) *NAND*s

(6 points)

Exercise 1.3:
Let $n \in \mathbb{N}$, $n \geq 7$. Prove that there exists a Boolean function $f : \{0,1\}^n \to \{0,1\}$ such that there exists no netlist realizing f with at most $\frac{2^{n-1}}{n}$ circuits, each with at most two inputs.

(4 points)

Deadline: Thursday, April 18th, before the lecture.

The websites for lecture and exercises are linked at

http://www.or.uni-bonn.de/lectures/ss13/ss13.html

In case of any questions feel free to contact me at rotter@or.uni-bonn.de or 73-8750