Models in Transportation

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Transportation Models

- large variety of models due to the many modes of transportation
  - roads
  - railroad
  - shipping
  - airlines
- as a consequence different type of equipment and resources with different characteristics are involved
  - cars, trucks, roads
  - trains, tracks and stations
  - ships and ports
  - planes and airports
- consider two specific problems

Tanker Scheduling

Basic Characteristics

- consider the problem from the view of a company
- the planning process normally is done in a 'rolling horizon' fashion
- company operates a fleet of ships consisting of
  - own ships $\{1, \ldots, T\}$
  - chartered ships
- the operating costs of these two types are different
- only the own ships are scheduled
- using chartered ships only leads to costs and these costs are given by the spot market

Tanker Scheduling

Basic Characteristics (cont.)

- each own ship i is characterized by its
    - capacity cap_i
    - draught dr_i
    - range of possible speeds
    - location l_i and time r_i at which it is ready to start next trip
    - ...

Basic Characteristics (cont.)

- the company has n cargos to be transported
- cargo j is characterized by
  - type t_j (e.g. crude type)
  - quantity p_j
  - load port $port_j^l$ and delivery port $port_j^d$
  - time windows $[r_j^l, d_j^l]$ and $[r_j^d, d_j^d]$ for loading and delivery
  - load and unload times t_j^l and t_j^d
  - costs c_j^* denoting the price which has to be paid on the spot market to transport cargo j (estimate)

Tanker Scheduling

Basic Characteristics (cont.)

- there are p different ports
- port k is characterized by
    - its location
    - limitations on the physical characteristics (e.g. length, draught, deadweight, ...) of the ships which may enter the port
    - local government rules (e.g. in Nigeria a ship has to be loaded above 90% to be allowed to sail)
    - ...

Basic Characteristics (cont.)

- the objective is to minimize the total cost of transporting all cargos
- hereby a cargo can be assigned to a ship of the company or 'sold' on the spot market and thus be transported by a chartered ship
- costs consist of
  - operating costs for own ships
  - spot charter rates
  - fuel costs
  - port charges, which depend on the deadweight of the ship

Tanker Scheduling

ILP modeling

- straightforward choice of variables would be to use $0-1$-variables for assigning cargos to ships
- problem: these assignment variables do not define the schedule/route for the ship and thus feasibility and costs of the assignment can not be determined
- alternative approach: generate a set of possible schedules/routes for each ship and afterwards use assignment variables to assign schedules/routes to ships
- problem splits up into two subproblems:
  - generate schedules for ships
  - assign schedules to ships

Tanker Scheduling

ILP modeling - generate schedules

- a schedule for a ship consist of an assignment of cargos to the ship and a sequence in which the corresponding ports are visited
- generation of schedules can be done by ad-hoc heuristics which consider
  - ship constraints like capacity, speed, availability, ...
  - port constraints
  - time windows of cargos
- each schedule leads to a certain cost
- for each ship enough potential schedules should be generated in order to get feasible and good solutions for the second subproblem

Tanker Scheduling

ILP modeling - generate schedules (cont.)

- the output of the first subproblem is
  - a set S_i of possible schedules for ship i
  - each schedule $l \in S_i$ is characterized by
    - a vector $(a^l_{i1}, \ldots, a^l_{in})$ where $a^l_{ij} = 1$ if cargo j is transported by ship i in schedule l and 0 otherwise
    - costs c^l_i denoting the incremental costs of operating ship i under schedule l versus keeping it idle over the planning horizon
    - profit $\pi^l_i = \sum_{j=1}^{n} a^l_{ij} c^*_j - c^l_i$ by using schedule l for ship i instead of paying the spot market

ILP modeling - generate schedules (cont.)

- Remarks:
  - all the feasibility constraints of the ports and ships are now within the schedule
  - all cost aspects are summarized in the values c_j^l resp. π_j^l
  - the sequences belonging to the schedules determine feasibility and the costs c_j^l but are not part of the output since they are not needed in the second subproblem

Tanker Scheduling

ILP modeling - assign schedules to ships

- variables $x_i^l = \begin{cases} 1 & \text{if ship } i \text{ follows schedule } l \\ 0 & \text{else} \end{cases}$
- objective: $\max \sum_{i=1}^{T} \sum_{l \in S_i} \pi_i^l x_i^l$
- constraint:
  - $\sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x_i^l \leq 1; \quad j = 1, \ldots, n$ (each cargo at most once)
  - $\sum_{l \in S_i} x_i^l \leq 1; \quad i = 1, \ldots, T$ (each ship at most one schedule)

Tanker Scheduling

ILP modeling - assign schedules to ships (cont.)

- the ILP model is a set-packing problem and well studied in the literature
- can be solved by branch and bound procedures
- possible branchings:
  - chose a variable x_i^l and branch on the two possibilities $x_i^l = 0$ and $x_i^l = 1$
    select x_i^l on base of the solution of the LP-relaxation: choose a variable with value close to 0.5
  - chose a ship i and branch on the possible schedules $l \in S_i$
    selection of ship i is e.g. be done using the LP-relaxation: choose a ship with a highly fractional solution

ILP modeling - assign schedules to ships (cont.)

- lower bounds can be achieved by generating feasible solutions via clever heuristics (feasible solution = lower bound since we have a maximization problem)
- upper bounds can be obtained via relaxing the integrality constraints and solving the resulting LP (note, that this LP-solution is also used for branching!)
- for a small example, the behavior of the branch and bound method is given in the handouts

Remarks Two Phase Approach

- in general the solution after solving the two subproblems is only a heuristic solution of the overall problem
- if in the first subproblem all possible schedules/routes for each ship are generated (i.e. S_i is equal to the set S_i^{all} of all feasible schedules for ship i), the optimal solution of the second subproblem is an optimal solution for the overall problem
- for real life instances the cardinalities of the sets S_i^{all} are too large to allow a complete generation (i.e. S_i is always a (small) subset of S_i^{all})
- colum generation can be used to improve the overall quality of the resulting solution

Train Timetabling

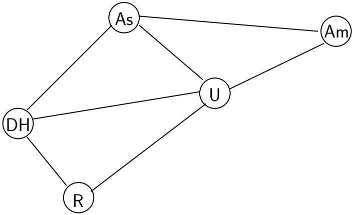
General Remarks

- in the railway world lots of scheduling problems are of importance
  - scheduling trains in a timetable
  - routing of material
  - staff planning
  - ...
- currently lots of subproblems are investigated
- the goal is to achieve an overall decision support system for the whole planning process
- we consider one important subproblem

Train Timetabling

Decomposition of the Train Timetabling

- mostly the overall railway network consists of some major stations and 'lines/corridors' connecting them



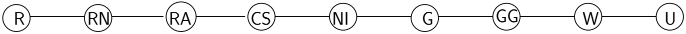
Am	Amersfoort
As	Amsterdam Centraal
DH	Den Haag Centraal
R	Rotterdam Centraal
U	Utrecht Centraal

- a corridor normally consists of two independent one-way tracks
- having good timetables for the trains in the corridors makes it often easy to find timetables for the trains on the other lines

Train Timetabling

Scheduling Train on a Track

- consider a track between two major stations
- in between the two mayor stations several smaller stations exists



R	R'dam Centraal	CS	Capelle Schollevaar	GG	Gouda Goverwelle
RN	R'dam Noord	NI	Nieuwerkerk ad IJssel	W	Woerden
RA	R'dam Alexander	G	Gouda	U	Utrecht Centraal

- trains may or may not stop at these stations
- trains can only overtake each other at stations

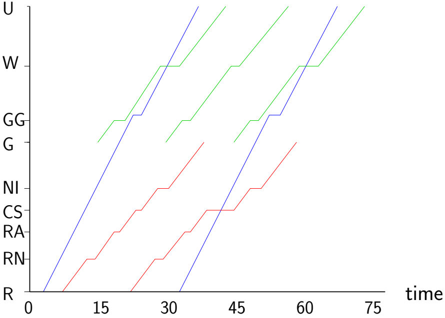
Train Timetabling

Problem Definition Track Scheduling

- time period $1, \ldots, q$, where q is the length of the planning period (typically measured in minutes; e.g. $q = 1440$)
- $L + 1$ stations $0, \ldots, L$
- L consecutive links;
- link j connects station $j - 1$ and j
- trains travel in the direction from station 0 to L
- T: set of trains that are candidates to run during planning period
- for link j, $T_j \subset T$ denotes the trains passing the link

Problem Definition Track Scheduling (cont.)

- train schedules are depicted in so-called time-space diagrams



- diagrams enable user to see conflicts

Problem Definition Track Scheduling (cont.)

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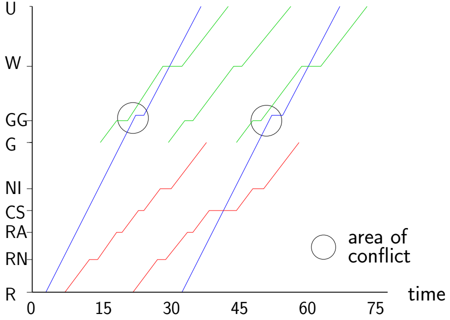


- diagrams enable user to see conflicts

Train Timetabling

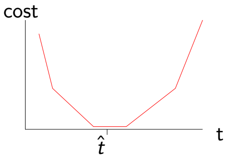
Problem Definition Track Scheduling (cont.)

- each train has an most desirable timetable (arrivals, departures, travel time on links, stopping time at stations), achieved e.g. via marketing department
- putting all these most desirable timetables together, surely will lead to conflicts on the track
- possibilities to change a timetable:
  - slow down train on link
  - increase stopping time at a station
  - modify departure time at first station
  - cancel the train

Train Timetabling

Problem Definition Track Scheduling (cont.)

- cost of deviating from a given time $\hat{t}$:
  - specifies the revenue loss due to a deviation from $\hat{t}$
  - the cost function has its minimum in $\hat{t}$, is convex, and often modeled by a piecewise linear function



- piecewise linear helps in ILP models!

Train Timetabling

Variables for Track Scheduling

- variables represent departure and arrival times from stations
  - y_{ij}: time train i enters link j

    $=$ time train i departs from station $j-1$

    (defined if $i \in T_j$)
  - z_{ij}: time train i leaves link j

    $=$ time train i arrives at station j

    (defined if $i \in T_j$)
- $c^d_{ij}(y_{ij})$ ($c^a_{ij}(z_{ij})$) denotes the cost resulting from the deviation of the departure time y_{ij} (arrival time z_{ij}) from its most desirable value

Variables for Track Scheduling (cont.)

- variables resulting from the departures and arrivals times:
  - $\tau_{ij} = z_{ij} - y_{ij}$: travel time of train i on link j
  - $\delta_{ij} = y_{i,j+1} - z_{ij}$: stopping time of train i at station j
- $c_{ij}^{\tau}(\tau_{ij})$ ($c_{ij}^{\delta}(\delta_{ij})$) denotes the cost resulting from the deviation of the travel time τ_{ij} (stopping time δ_{ij}) from its most desirable value
- all cost functions $c_{ij}^{d}, c_{ij}^{a}, c_{ij}^{\tau}, c_{ij}^{\delta}$ have the mentioned structure

Train Timetabling

Objective function

- minimize

$$\sum_{j=1}^{L}\sum_{i\in T_j}(c_{ij}^{d}(y_{ij}) + c_{ij}^{a}(z_{ij}) + c_{ij}^{\tau}(z_{ij} - y_{ij})) + \sum_{j=1}^{L-1}\sum_{i\in T_j} c_{ij}^{\delta}(y_{i,j+1} - z_{ij})$$

Train Timetabling

Constraints

- minimum travel times for train i over link j: τ_{ij}^{min}
- minimum stopping times for train i at station j: δ_{ij}^{min}
- safety distance:
  - minimum headway between departure times of train h and train i from station j: H_{hij}^{d}
  - minimum headway between arrival times of train h and train i at station j: H_{hij}^{a}
- lower and upper bounds on departure and arrival times: $y_{ij}^{min}, y_{ij}^{max}, z_{ij}^{min}, z_{ij}^{max}$

Constraints (cont.)

- to be able to model the minimum headway constraints, variables have to be introduced which control the order of the trains on the links
- $x_{hij} = \begin{cases} 1 & \text{if train } h \text{ immediately preceeds train } i \text{ on link } j \\ 0 & \text{else} \end{cases}$
- using the variables x_{hij}, the minimum headway constraints can be formulated via 'big M'-constraints:

$$y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M \geq H^d_{hij}$$

$$z_{ij} - z_{hj} + (1 - x_{hij})M \geq H^a_{hij}$$

Constraints (cont.)

- two dummy trains 0 and $*$ are added, representing the start and end of the planning period (fix departure and arrival times appropriate ensuring that 0 is sequenced before all other trains and $*$ after all other trains)

Constraints (cont.)

$$
\begin{aligned}
y_{ij} &\geq y_{ij}^{min} & j &= 1, \ldots, L;\ i \in T_j \\
y_{ij} &\leq y_{ij}^{max} & j &= 1, \ldots, L;\ i \in T_j \\
z_{ij} &\geq z_{ij}^{min} & j &= 1, \ldots, L;\ i \in T_j \\
z_{ij} &\leq z_{ij}^{max} & j &= 1, \ldots, L;\ i \in T_j \\
z_{ij} - y_{ij} &\geq \tau_{ij}^{min} & j &= 1, \ldots, L;\ i \in T_j \\
y_{i,j+1} - z_{ij} &\geq \delta ij^{min} & j &= 1, \ldots, L-1;\ i \in T_j \\
y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M &\geq H_{hij}^{d} & j &= 0, \ldots, L-1;\ i, h \in T_j \\
z_{ij} - z_{hj} + (1 - x_{hij})M &\geq H_{hij}^{a} & j &= 1, \ldots, L;\ i, h \in T_j \\
\textstyle\sum_{h \in T_j \setminus \{i\}} x_{hij} &= 1 & j &= 1, \ldots, L;\ i \in T_j \\
\textstyle\sum_{i \in T_j \setminus \{h\}} x_{hij} &= 1 & j &= 1, \ldots, L;\ h \in T_j \\
x_{hij} &\in \{0, 1\} & j &= 1, \ldots, L;\ i, h \in T_j
\end{aligned}
$$

Remarks on ILP Model

- the number of 0-1 variables gets already for moderate instances quite large
- the single track problem is only a subproblem in the whole time tabling process and needs therefore to be solved often
- as a consequence, the computational time for solving the single track problem must be small
- this asks for heuristic approaches to solve the single track problem

Train Timetabling

Decomposition Approach: General Idea

- schedule the trains iteratively one by one
- initially, the two dummy trains 0 and $*$ are scheduled
- the selection of the next train to be scheduled is done on base of priorities
- possible priorities are
  - earliest desired departure time
  - decreasing order of importance (importance may be e.g. measured by train type, speed, expected revenue, ...)
  - smallest flexibility in departure and arrival
  - combinations of the above

Decomposition Approach: Realization

- T_0: set of already scheduled trains
- initially $T_0 = \{0, *\}$
- after each iteration a schedule of the trains from T_0 is given
- however, for the next iteration only the sequence in which the trains from T_0 traverse the links is taken into account
- $S_j = (0 = j_0, j_1, \ldots, j_{n_j}, j_{n_j+1} = *)$: sequence of trains from T_0 on link j
- if train k is chosen to be scheduled in an iteration, we have to insert k in all sequences S_j where $k \in T_j$
- this problem is called $\mathit{Insert}(k, T_0)$

ILP Formulation of *Insert*(k, T_0)

Adapt the 'standard' constraints and the objective to T_0:

$$\min \sum_{j=1}^{L} \sum_{i \in T_j} (c_{ij}^d(y_{ij}) + c_{ij}^a(z_{ij}) + c_{ij}^\tau(z_{ij} - y_{ij}))$$
$$+ \sum_{j=1}^{L_1} \sum_{i \in T_j} c_{ij}^\delta(y_{i,j+1} - z_{ij})$$

subject to

$$\begin{aligned}
y_{ij} &\geq y_{ij}^{min} & & j = 1, \ldots, L;\ i \in T_0 \cap T_j \\
y_{ij} &\leq y_{ij}^{max} & & j = 1, \ldots, L;\ i \in T_0 \cap T_j \\
z_{ij} &\geq z_{ij}^{min} & & j = 1, \ldots, L;\ i \in T_0 \cap T_j \\
z_{ij} &\leq z_{ij}^{max} & & j = 1, \ldots, L;\ i \in T_0 \cap T_j \\
z_{ij} - y_{ij} &\geq \tau_{ij}^{min} & & j = 1, \ldots, L;\ i \in T_0 \cap T_j \\
y_{i,j+1} - z_{ij} &\geq \delta ij^{min} & & j = 1, \ldots, L-1;\ i \in T_0 \cap T_j
\end{aligned}$$

ILP Formulation of $\mathit{Insert}(k, T_0)$ (cont.)

- adapt $y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M \geq H^d_{hij}$ for trains from T_0

$$y_{j_{i+1},j} - y_{j_i,j} \geq H^d_{j_i j_{i+1},j-1} \quad \text{for } j = 1, \ldots, L, i = 0, \ldots, n_j$$

- adapt $z_{ij} - z_{hj} + (1 - x_{hij})M \geq H^a_{hij}$ for trains from T_0

$$z_{j_{i+1},j} - z_{j_i,j} \geq H^a_{j_i j_{i+1} j} \quad \text{for } j = 1, \ldots, L, i = 0, \ldots, n_j$$

<u>ILP Formulation of *Insert*(k, T_0) (cont.)</u>

- insert k on link j via variables

$$x_{ij} = \begin{cases} 1 & \text{if train } k \text{ immediately precedes train } j_i \text{ on link } j \\ 0 & \text{else} \end{cases}$$

- new constraints for $j = 1, \ldots, L, i = 0, \ldots, n_j$:
  - $y_{k,j} - y_{j_i,j} + (1 - x_{ij})M \geq H^d_{j_i kj}$
  - $y_{j_{i+1},j} - y_{k,j} + (1 - x_{ij})M \geq H^d_{kj_{i+1}j}$
  - $z_{k,j} - z_{j_i,j} + (1 - x_{ij})M \geq H^a_{j_i kj}$
  - $z_{j_{i+1},j} - z_{k,j} + (1 - x_{ij})M \geq H^a_{kj_{i+1}j}$
- 0-1 constraints and sum constraint on x_{ij} values

Remarks on ILP Formulation of $\mathit{Insert}(k, T_0)$

- the ILP Formulation of $\mathit{Insert}(k, T_0)$ has the same order of continuous constraints (y_{ij}, z_{ij}) but far fewer 0-1 variables than the original MIP
- a preprocessing may help to fix x_{ij} variables since on base of the lower and upper bound on the departure and arrival times of train k many options may be impossible
- solving $\mathit{Insert}(k, T_0)$ may be done by branch and bound

Solving the overall problem

- an heuristic for the overall problem may follow the ideas of the shifting bottleneck heuristic
  - select a new train k (machine) which is most 'urgent'
  - solve for this new train k the problem $Insert(k, T_0)$
  - reoptimize the resulting schedule by rescheduling the trains from T_0
- rescheduling of a train $l \in T_0$ can be done by solving the problem $Insert(l, T_0 \cup \{k\} \setminus \{l\})$ using the schedule which results from deleting train l from the schedule achieved by $Insert(k, T_0)$