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On a conjecture of Hoàng and Tu concerning perfectly orderable graphs

STEFAN HOUGARDY

Humboldt-Universität zu Berlin
Institut für Informatik
Unter den Linden 6, 10099 Berlin, Germany
hougardy@informatik.hu-berlin.de

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Abstract. Hoàng and Tu [5] conjectured that a weakly triangulated graph which does not contain a chordless path with six vertices is perfectly orderable. We present a counterexample to this conjeture.

Keywords: perfectly orderable graph, weakly triangulated graph

A graph is called *perfectly orderable* if it admits a linear order "$<$" on its vertices such that no chordless path with four vertices a, b, c, d and edges ab, bc, cd has $a < b$ and $d < c$. The notion of perfectly orderable graphs has been introduced by Chvátal [1]. A graph is called *weakly triangulated* [3] if neither the graph nor its complement contains a chordless cycle of length at least 5.

In 1989 Chvátal [2] conjectured that a weakly triangulated graph which does not contain a chordless path with five vertices is perfectly orderable. This conjecture was proved by Hayward [4] in 1997.

Theorem 1 (Hayward [4]) *A weakly triangulated graph which does not contain a chordless path with five vertices is perfectly orderable.*

In 2000 Hoàng and Tu proposed the following natural extension of this result:

Conjecture 1 (Hoàng, Tu [5]) *A weakly triangulated graph which does not contain a chordless path with six vertices is perfectly orderable.*

We will present a counterexample to this conjecture in the following.

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Lemma 1 *There exist weakly triangulated graphs without a chordless path with six vertices that are not perfectly orderable.*

Proof. We will prove that the graph shown in Figure 1 has the desired properties. It is easy to see that this graph does not contain a chordless path with six vertices and that it is weakly triangulated. It remains to show that the graph is not perfectly orderable.

We will denote a chordless path with four vertices a, b, c, d and edges ab, bc, cd simply by $abcd$. Because of the symmetry of the graph we may assume $e < d$ without loss of generality. Now the chordless path $edih$ implies $i < h$. Next the chordless path $ihge$ implies $g < e$. The triangle deg must be acyclic, so $g < d$ must hold. Next the chordless path $gdac$ implies $a < c$. The chordless path $acfg$ implies $f < g$. Now the triangle egf must be acyclic, so $f < e$. The chordless path $feab$ implies $a < b$. Next the chordless path $abgf$ implies $g < f$. This is a contradiction as we already have $f < g$. □

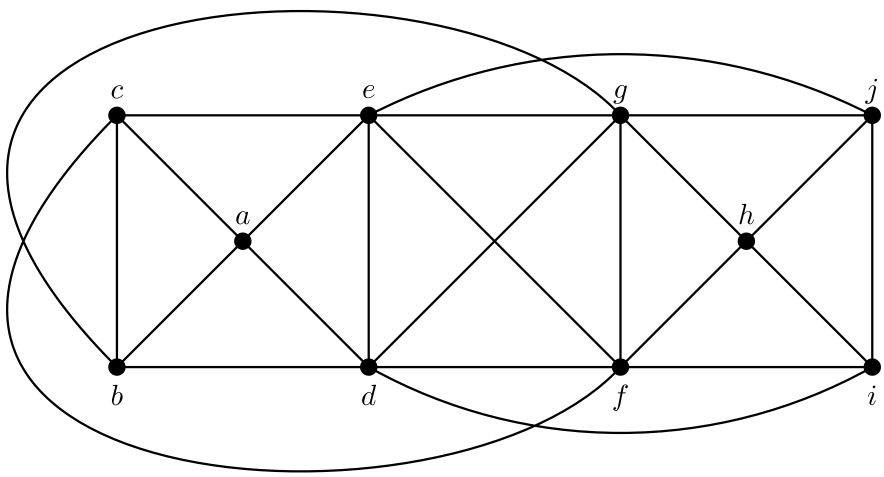

Figure 1: A counterexample to the conjecture of Hoàng and Tu.

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