This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Classes of Perfect Graphs

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February 28, 2003 revised October 2003, February 2005, and July 2007

Abstract. The Strong Perfect Graph Conjecture, suggested by Claude Berge in 1960, had a major impact on the development of graph theory over the last forty years. It has led to the definitions and study of many new classes of graphs for which the Strong Perfect Graph Conjecture has been verified. Powerful concepts and methods have been developed to prove the Strong Perfect Graph Conjecture for these special cases. In this paper we survey 120 of these classes, list their fundamental algorithmic properties and present all known relations between them.

1 Introduction

A graph is called *perfect* if the chromatic number and the clique number have the same value for each of its induced subgraphs. The notion of perfect graphs was introduced by Berge [6] in 1960. He also conjectured that a graph is perfect if and only if it contains, as an induced subgraph, neither an odd cycle of length at least five nor its complement.

This conjecture became known as the *Strong Perfect Graph Conjecture* and attempts to prove it contributed much to the developement of graph theory in the past forty years. The methods developed and the results proved have their uses also outside the area of perfect graphs. The theory of antiblocking polyhedra developed by Fulkerson [37], and the theory of modular decomposition (which has its origins in a paper of Gallai [39]) are two such examples.

The Strong Perfect Graph Conjecture has led to the definitions and study of many new classes of graphs for which the correctness of this conjecture has been verified. For several of these classes the Strong Perfect Graph Conjecture has been proved by showing that

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every graph in this class can be obtained from certain simple perfect graphs by repeated application of perfection preserving operations. By using this approach Chudnovsky, Robertson, Seymour and Thomas [19] were recently able to prove the Strong Perfect Graph Conjecture in its full generality. After remaining unsolved for more than forty years it can now be called the Strong Perfect Graph Theorem.

The aim of this paper is to survey 120 classes of perfect graphs. The criterion we used to include a class of perfect graphs in this survey is that its study be motivated by making progress towards a proof of the Strong Perfect Graph Conjecture. This criterion rules out including classes of perfect graphs that are known to be perfect just by definition, e.g. classes that are defined as subclasses of graphs already known to be perfect or classes that are defined as the union of two classes of perfect graphs. Some exceptions are made. For example we include some very basic classes such as trees or bipartite graphs. We have also included a few classes which were not known to contain only perfect graphs without using the Strong Perfect Graph Theorem. On the other hand, there probably exist several classes of perfect graphs which satisfy our criterion, but which are not included in this survey. We refer to [12, 13] for further information on graph classes.

A second motivation for studying perfect graphs besides the Strong Perfect Graph Conjecture are their nice algorithmic properties. While the problems of finding the clique number or the chromatic number of a graph are NP-hard in general, they can be solved in polynomial time for perfect graphs. This result is due to Grötschel, Lovász and Schrijver [47] from 1981. Unfortunately, their algorithms are based on the ellipsoid method and are therefore mostly of theoretical interest. It is still an open problem to find a combinatorial polynomial time algorithm to color perfect graphs or to compute the clique number of a perfect graph. However, for many classes of perfect graphs, such algorithms are known. In Section 4 we survey results of this kind. Moreover we consider the recognition complexity of all these classes, i.e. the question of deciding whether a given graph belongs to the class. Chudnovsky, Cornuejols, Liu, Seymour and Vušković [18] recently proved that there exists a polynomial time algorithm for recognizing perfect graphs. For several subclasses of perfect graphs such an algorithm is not yet known.

In many cases new classes of perfect graphs that have been introduced were motivated by generalizing known classes of perfect graphs. Many classes of perfect graphs are, therefore, subclasses of other classes of perfect graphs. We study the relation between all the classes of perfect graphs contained in this survey. The relations are given in the form of a table either stating that class A is contained in a class B or by giving an example of a graph showing that A is not a subclass of B. The table containing this information has 14400 entries. For several cases which had been open, the table answers the question whether a class A is a subclass of a class B.

The paper is organized as follows: Section 2 contains all basic notations used through-

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out this paper. The definitions of the classes of perfect graphs appearing in this paper are given in Section 3. In Section 4 we survey algorithms for the recognition and for solving optimization problems on classes of perfect graphs. The number of graphs contained in each of the classes of perfect graphs considered is given in Section 5. The relations between the classes of perfect graphs studied in this paper are presented in Section 6. All counterexamples that are needed to prove that certain classes are not contained in each other are described in Section 7.

2 Notation

Given a graph $G = (V, E)$ with vertex set V and edge set E we denote by n and m the cardinality of V and E. The *degree* of a vertex is the number of edges incident to this vertex. The *maximum degree* $\Delta(G)$ is the largest degree of a vertex of G. A *k-coloring* of the vertices of a graph $G = (V, E)$ is a map $f : V \to \{1, \ldots, k\}$ such that $f(x) \neq f(y)$ whenever $\{x, y\}$ is an edge in G. The *chromatic number* $\chi(G)$ is the least number k such that G admits a k-coloring. A *clique* is a graph containing all possible edges. A clique on i vertices is denoted by K_i. The *clique number* $\omega(G)$ of a graph G is the size of a largest clique contained in G as a subgraph. A *stable set* in a graph is a set of vertices no two of which are adjacent. By I_i we denote a stable set of size i. The *stability number* $\alpha(G)$ is the size of a largest stable set in G. The *complement* $\overline{G}$ of a graph G has the same vertex set as G and two vertices in $\overline{G}$ are adjacent if and only if they are not adjacent in G. Obviously, we have $\alpha(G) = \omega(\overline{G})$, and the *clique covering number* $\theta(G)$ is defined as $\chi(\overline{G})$.

A graph is called *perfect* if $\chi(H) = \omega(H)$ for every induced subgraph H. A *hole* is a chordless cycle of length at least four and an *antihole* is the complement of a hole. An *odd* (respectively *even*) hole is a hole with an odd (respectively even) number of vertices. A graph is called *Berge* if it contains no odd holes and no odd antiholes as induced subgraphs. A *star-cutset* in a graph G is a subset C of vertices such that $G \setminus C$ is disconnected and such that some vertex in C is adjacent to all other vertices in C.

A complete bipartite graph, i.e. a bipartite graph with all possible edges between the vertices of the two color classes of size r and s, respectively, is denoted by $K_{r,s}$. A $K_{1,3}$ is called a *claw*. A path on i vertices is denoted by P_i and a cycle on i vertices by C_i. The two vertices of degree one in a path are called the *endpoints* of the path. In a P_4 the vertices of degree two are called *midpoints* of the P_4. The two edges of a P_4 incident to the endpoints of the P_4 are called *wings*. The *wing graph* $W(G)$ of a graph G has as its vertices all edges of G and two edges are adjacent in $W(G)$ if there is an induced P_4 in G that has these two edges as its wings. Given a graph G its *k-overlap graph* is

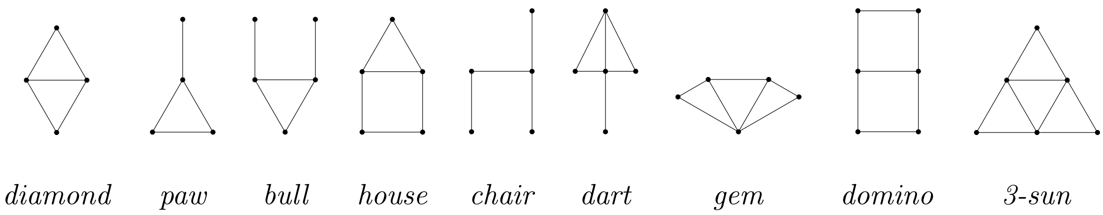

Figure 1: Some small graphs with special names.

defined as the graph whose vertices are all induced P_4's of G and in which two vertices are adjacent if the corresponding P_4's in G have exactly k vertices in common. Two vertices x, y in a graph are called *partners* if there exist vertices u, v, w distinct from x, y such that $\{x, u, v, w\}$ and $\{y, u, v, w\}$ each induce a P_4 in the graph. The *partner graph* of a graph G is the graph whose vertices are the vertices of G and whose edges join pairs of partners in G.

Two vertices form an *even pair* if all induced paths between these two vertices have even length. The *line graph* $L(G)$ of a graph G is the graph that has the edges of G as vertices and in which two vertices in $L(G)$ are adjacent if the corresponding edges of G are adjacent (that is, share a vertex). Some small graphs are given special names. Figure 1 contains such graphs with the names that are used throughout this paper.

3 Definitions of Graph Classes

In this section we briefly present in alphabetical order the definitions of all classes of perfect graphs appearing in this paper. For each class we give a reference to a proof that all graphs in the class are perfect. Note that with the proof of the Strong Perfect Graph Conjecture it follows immediately for all classes that they contain only perfect graphs.

alternately colorable A graph is called alternately colorable if its edges can be colored using only two colors in such a way that in every induced cycle of length at least four no two adjacent edges have the same color. This class of graphs has been defined by Hoàng [61] who also proved the perfectness of these graphs.

alternately orientable A graph is called alternately orientable if it admits an orientation of its edges such that in every induced cycle of length at least four the orientation of the edges alternates. This class of graphs was defined by Hoàng [61] who also proved the perfectness of these graphs.

AT-free Berge A graph is called AT-free Berge if it is a Berge graph and does not

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

contain an asteroidal triple. An asteroidal triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third. This class of graphs was introduced in [80]. Perfectness of these graphs was observed by Maffray [29, page 401]. As his argument is unpublished we briefly state it here. If an AT-free Berge graph has stability number two then it must be the complement of a bipartite graph and therefore perfect. If the graph has a stable set of size three, say $\{x, y, z\}$, then since the graph is AT-free it must be that the set of all neighbours of one of them, say z, separates x from y, i.e., z is the center of a star-cutset. Perfection follows from [21].

BIP* A graph belongs to the class BIP* if all induced subgraphs H which are not bipartite have the property that H or $\overline{H}$ contains a star–cutset. This class of graphs was defined by Chvátal [21] who also proved the perfectness of these graphs.

bipartite A graph is called bipartite if its chromatic number is at most two. Perfectness of bipartite graphs follows from the definition.

brittle A graph is called brittle if every induced subgraph H of G contains a vertex that is not an endpoint or not a midpoint of a P_4 in H. This class of graphs was introduced by Chvátal. Perfection follows easily as all brittle graphs are perfectly orderable [63].

bull-free Berge A bull-free Berge graph is a Berge graph that does not contain a bull (see Figure 1) as an induced subgraph. Chvátal and Sbihi [24] proved that these graphs are perfect.

C_4**-free Berge** A C_4-free Berge graph is a Berge graph that does not contain a cycle on four vertices as an induced subgraph. Perfection of these graphs was shown by Conforti, Cornuéjols, and Vušković [28].

chair-free Berge A chair-free Berge graph is a Berge graph that does not contain a chair (see Figure 1) as an induced subgraph. Perfection of these graphs was shown by Sassano [107].

chordal see →*triangulated*.

claw-free Berge A graph is claw-free Berge if it is a Berge graph that does not contain a $K_{1,3}$ (which is called a claw) as an induced subgraph. Parthasarathy and Ravindra [96] proved the perfectness of these graphs.

clique-separable A graph is called clique-separable if every induced subgraph that does not contain a clique-cutset is of one of the following two types. Either it is a complete multipartite graph or its vertex set can be partitioned into two sets V_1

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and V_2 such that V_1 is a connected bipartite graph, V_2 is a clique and all vertices in V_1 are connected to all vertices in V_2. This class of graphs appears first in the paper of Gallai [38]. Gavril [41] invented the name for this class. Perfection follows immediately from the definition.

co-*class* Complements of the graphs in →*class*.

cograph see →P_4*-free*.

cograph contraction A graph G is a cograph contraction if there exists a cograph H and some pairwise disjoint independent sets in H such that G is obtained from H by contracting each of the independent sets to a single vertex (resulting multiple edges are identified) and joining the new vertices pairwise. Hujter and Tuza [73] introduced this class of graphs and proved that they are perfect. A good characterization of these graphs is given in [79].

comparability A graph is a comparability graph if there exists a partial order "$<$" on its vertices such that two vertices x and y are adjacent in the graph if and only if $x < y$ or $y < x$. These graphs are also called *transitively orientable*. Perfectness follows from a classical result of Dilworth [33].

$\Delta \leq 6$ **Berge** The class $\Delta \leq 6$ Berge contains all Berge graphs in which the maximum degree is at most 6. Grinstead [46] proved that these graphs are perfect.

dart-free Berge A graph is dart-free Berge if it is a Berge graph that does not contain a dart (see Figure 1) as an induced subgraph. Sun [114] proved the perfectness of these graphs.

degenerate Berge A graph is called degenerate Berge if it is a Berge graph and every induced subgraph H has a vertex of degree at most $\omega(H) + 1$. This class of graphs has been defined by Aït Haddadène and Maffray [1] who also proved the perfectness of these graphs.

diamond-free Berge A graph is diamond-free Berge if it is a Berge graph that does not contain a diamond (a K_4 with one edge removed, see Figure 1) as an induced subgraph. Tucker [119] proved the perfectness of these graphs based on earlier results of Parthasarathy and Ravindra [97].

doc-free Berge The name doc-free Berge is an abbreviation for the class of diamonded odd cycle-free Berge graphs. These are Berge graphs that do not contain diamonded odd cycles as induced subgraphs. A diamonded odd cycle on five vertices is a P_4 or a C_4 together with a fifth vertex joined to all the others. An odd cycle C with more than five vertices is called a diamonded odd cycle if it has two chords $\{x, y\}$

and $\{x, z\}$ with $\{y, z\}$ an edge of C and there exists a vertex w not on C adjacent to y and z but not x. Moreover no edge of C other than $\{y, z\}$ is on a triangle induced by the vertices of C. Carducci [17] proved the perfectness of doc-free Berge graphs.

elementary A graph is called elementary if its edges can be colored by two colors so that no monochromatic induced P_3 occurs. Equivalently these are graphs whose Gallai-graph is bipartite. Elementary graphs were introduced by Chvátal and Sbihi [25]. Perfectness of these graphs follows from the fact that they are claw-free Berge. Maffray and Reed [84] give a description of the structure of elementary graphs.

forest A graph is called a forest if it does not contain a cycle. These graphs are perfect as they are bipartite.

Gallai There exist two different classes of perfect graphs which have been given the name Gallai. Historically → *triangulated graphs* were called Gallai graphs [9]. Later, → *i-triangulated graphs* were given this name.

gem-free Berge A graph is called gem-free Berge if it is a Berge graph without a gem (see Figure 1) as an induced subgraph. Perfection of these graphs follows from the Strong Perfect Graph Theorem [19].

HHD-free A graph is called HHD-free if it does not contain a house (see Figure 1), a hole of length at least 5 or a domino (see Figure 1) as an induced subgraph. This class of graphs was introduced in [63]. Perfectness follows easily from the observation that these graphs are Meyniel.

Hoàng A graph is called Hoàng if its wing graph (see Section 2) is bipartite. This class of graphs was introduced in [22]. Perfection of these graphs follows from the Strong Perfect Graph Theorem [19].

***i*-triangulated** A graph is called i-triangulated if every odd cycle of length at least five has two non-crossing chords. These graphs are also called →*Gallai*. Gallai [38] proved the perfectness of these graphs.

I_4-free Berge A graph is I_4-free Berge if it is Berge and does not contain a stable set on four vertices. These are complements of → K_4-free Berge graphs.

interval A graph is an interval graph if each vertex can be represented by an interval on the real line in such a way that two vertices are adjacent if and only if their corresponding intervals intersect. These graphs are → *triangulated* [43] and therefore perfect.

K_4-free Berge A graph is K_4-free Berge if it is Berge and does not contain a clique on four vertices. Tucker [118] proved the perfectness of these graphs.

(K_5, P_5)-free Berge A graph is (K_5, P_5)-free Berge if it is Berge and does not contain a K_5 or a P_5 as an induced subgraph. Perfectness of these graphs was proved by Maffray and Preissmann [82].

LGBIP The class LGBIP consists of all line graphs (see Section 2) of bipartite graphs. As noted in [6] perfection of these graphs follows from a classical result of König [78].

line perfect A graph is called line perfect if its line graph is perfect. Perfection of these graphs follows from a characterization of Trotter [116].

locally perfect A graph is called locally perfect if every induced subgraph admits a coloring of its vertices such that for any vertex the number of colors used in the neighborhood of this vertex equals the clique number of the neighborhood of the vertex. This class of graphs was introduced by Preissmann [98] who also proved the perfection of these graphs.

Meyniel A graph is called Meyniel if every odd cycle of length at least five has at least two chords. Meyniel [87, 88] proved the perfectness of these graphs. The same result was proven independently by Markosian and Karapetian [86].

murky A graph is called murky if it contains no C_5, P_6 or $\overline{P}_6$ as an induced subgraph. Hayward [52] proved that murky graphs are perfect.

1-overlap bipartite A graph belongs to the class 1-overlap bipartite if it is C_5-free and its 1-overlap graph (see Section 2) is bipartite. Hoàng, Hougardy and Maffray [62] proved that these graphs are perfect.

opposition A graph is called opposition if it admits an orientation of its edges such that in every induced P_4 the two end edges both either point inwards or outwards. This class of graphs was introduced by Chvátal [22]. Perfection follows from the Strong Perfect Graph Theorem [19]. Note that there is another class of perfect graphs called opposition [92] which additionally requires that the orientation of the edges be acyclic. Therefore we call this class → *strict opposition*.

P_4-free A graph is called P_4-free if it does not contain a P_4 as an induced subgraph. These graphs are also called cographs. Perfection follows from a result of Seinsche [110].

P_4-lite A graph is called P_4-lite if every induced subgraph H with at most six vertices contains either at most two induced P_4's or H or $\overline{H}$ is the 3-sun (see Section 2). These graphs were introduced in [76]. Perfection follows from the fact that they are → *weakly triangulated*.

P_4-reducible A graph is called P_4-reducible if every vertex belongs to at most one

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

induced P_4. These graphs were introduced in [75]. Perfection follows from the fact that they are → *weakly triangulated.*

P_4-sparse A graph is called P_4-sparse if no set of five vertices induces more than one P_4. This class of graphs was introduced in [60]. Perfection follows from the fact that these graphs are → *weakly triangulated.*

P_4-stable Berge A graph is called P_4-stable Berge if it is a Berge graph containing a stable set that intersects all induced P_4's. Hoàng and Le [64] proved that these graphs are perfect.

parity A graph is called parity if for every pair of nodes, the lengths of all induced paths connecting them have the same parity. Burlet and Uhry [16] proved that a graph is parity if and only if each odd cycle of length at least five has two crossing chords. Perfection of these graphs was proved by Olaru [94].

partner-graph triangle-free The class partner-graph triangle-free contains all graphs whose partner graph (see Section 2) is triangle free. Perfection of this class of graphs was proved by Hayward and Lenhart [54].

paw-free Berge A graph is called paw-free Berge if it is a Berge graph that does not contain a paw (see Figure 1) as an induced subgraph. Perfection follows from the observation that these graphs are Meyniel. See [93] for a characterization of paw-free graphs.

perfectly contractile A graph is called perfectly contractile if for any induced subgraph H there exists a sequence $H = H_0, H_1, \dots, H_k$ for some k such that H_{i+1} is obtained from H_i by contraction of an even pair (see Section 2) and H_k is a clique. Bertschi [10] introduced this class of graphs and proved that they are perfect.

perfectly orderable A graph is called perfectly orderable if there exists an acyclic orientation of the edges such that in no induced P_4 the two end edges are oriented inwards. This class of graphs was introduced by Chvátal [20] who also proved that they are perfect.

permutation A graph is called a permutation graph if it can be represented by a permutation $\pi : \{1, \dots, n\} \to \{1, \dots, n\}$ in such a way that two vertices $i < j$ are adjacent if and only if $\pi(i) > \pi(j)$. Perfection of these graphs follows from a characterization of Dushnik and Miller [35].

planar Berge The class planar Berge contains all Berge graphs that are planar. Perfection of these graphs was shown by Tucker [117].

preperfect A vertex x in a graph G is called predominant if there exists another vertex

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y such that every maximum clique of G containing y contains x or every maximum stable set containing x contains y. A graph is called preperfect if every induced subgraph has a predominant vertex. Hammer and Maffray [49] introduced this class of graphs and proved that all preperfect graphs are perfect.

quasi-parity A graph is called quasi-parity if for every induced subgraph H of G either H or $\overline{H}$ contains an even pair (see Section 2). Meyniel [89] proved that quasi-parity graphs are perfect.

Raspail A graph is called Raspail if every odd cycle has a short chord, i.e. a chord joining two vertices that have distance two on the cycle. See [114] for an explanation of where the name for this class comes from. Perfection of these graphs follows from the Strong Perfect Graph Theorem [19].

skeletal A graph is called skeletal if it can be obtained by removing a collection $\mathcal{S}$ of stars in a → *parity* graph. No two centers of stars in $\mathcal{S}$ must be joined by an induced path of length at most two. Hertz [58] proved that these graphs are perfect.

slender A graph is called slender if it can be obtained from an →*i-triangulated* graph by deleting all the edges of an arbitrary matching. Hertz [57] proved that these graphs are perfect.

slightly triangulated A graph is called slightly triangulated if it contains no hole of length at least five and every induced subgraph H contains a vertex whose neighborhood in H does not contain a P_4. This class of graphs was introduced by Maire [85] who also proved the perfectness of these graphs.

slim A graph is called slim if it can be obtained from a Meyniel graph by removing all the edges that are induced by an arbitrary vertex set. Hertz [56] proved that slim graphs are perfect.

snap A graph is called snap if it is Berge and every induced subgraph contains a vertex whose neighborhood can be partitioned into a stable set and a clique. Maffray and Preissmann [83] proved the perfection of snap graphs.

split A graph is called split if its vertex set can be partitioned into two sets V_1 and V_2 such that V_1 induces a stable set and V_2 induces a clique. Perfection of split graphs follows from the fact that they are triangulated.

strict opposition A graph is called strict opposition if it admits an acyclic orientation of its edges such that in every induced P_4 the two end edges both either point inwards or outwards. Olariu [92] proved that these graphs are perfect.

strict quasi-parity A graph is called strict quasi-parity if every induced subgraph

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either contains an even pair (see Section 2) or is a clique. Meyniel [89] proved that strict quasi-parity graphs are perfect.

strongly perfect A graph is called strongly perfect if every induced subgraph contains a stable set that intersects all maximal cliques. Berge and Duchet [8] introduced strongly perfect graphs and proved their perfection.

3-overlap bipartite A graph belongs to the class 3-overlap bipartite if its 3-overlap graph (see Section 2) is bipartite. Hoàng, Hougardy and Maffray [62] proved that these graphs are perfect.

3-overlap triangle free A graph belongs to the class 3-overlap bipartite if it is Berge and its 3-overlap graph (see Section 2) is triangle free. Hoàng, Hougardy and Maffray [62] proved that these graphs are perfect.

threshold A graph is called a threshold graph if it does not contain a C_4, $\overline{C}_4$ and P_4 as an induced subgraph. Perfection of these graphs follows easily as they are triangulated.

totally unimodular see →*unimodular*.

transitively orientable see →*comparability*.

tree A connected graph that does not contain a cycle is called a tree. Trees are perfect as they are bipartite.

triangulated A graph is called triangulated if every cycle of length at least four contains a chord. These graphs are also called *chordal*. Perfection of triangulated graphs follows from results of Hajnal and Surányi [48] and Dirac [34].

trivially perfect A graph is called trivially perfect if for each induced subgraph H the stability number of H equals the number of maximal cliques in H. Golumbic [44] introduced these graphs and proved their perfection. He also showed that a graph is trivially perfect if and only if it contains no C_4 and no P_4 as an induced subgraph.

2-overlap bipartite A graph belongs to the class 2-overlap bipartite if it is C_5-free and its 2-overlap graph (see Section 2) is bipartite. Hoàng, Hougardy and Maffray [62] proved that these graphs are perfect.

2-overlap triangle free A graph belongs to the class 2-overlap triangle-free if it is Berge and its 2-overlap graph (see Section 2) is triangle free. Hoàng, Hougardy and Maffray [62] proved that these graphs are perfect.

2-split Berge A graph is called 2-split Berge if it is a Berge graph and if it can be

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partioned into two → *split* graphs. Hoàng and Le [65] proved that 2-split graphs are perfect.

$2K_2$-free Berge These are the complements of → *C_4-free Berge graphs.*

unimodular A graph is called unimodular if its incidence matrix of vertices and maximal cliques is totally unimodular, i.e. every square submatrix has determinant 0, 1, or −1. Perfection of these graphs was proved by Berge [7].

weakly chordal see →*weakly triangulated.*

weakly triangulated A graph is called weakly triangulated if neither the graph nor its complement contains an induced cycle of length at least five. These graphs are also called *weakly chordal.* Hayward [51] proved that weakly triangulated graphs are perfect.

wing triangulated A graph is called wing triangulated if its wing graph (see Section 2) is triangulated. Hougardy, Le and Wagler [68] proved that wing triangulated graphs are perfect.

4 Algorithmic Complexity

The following table lists what is known regarding algorithmic complexity for the 120 classes. Note that we do not include the complements of the classes as they have, except in the case of linear time recognition, the same algorithmic behavior as the classes themselves. The column *recognition* contains information on polynomial time algorithms to test whether a given graph is a member of the class. The columns ω, χ, α, and θ contain information on polynomial time *combinatorial* algorithms to compute a maximum clique, the chromatic number, the stability number or a clique covering. Note that all these problems can be solved in polynomial time by the algorithms of Grötschel, Lovász, and Schrijver [47]. However, their algorithms are based on the ellipsoid method and are therefore not purely combinatorial.

We use the following notation in the table: P means there exists a polynomial time algorithm but we do not specify its running time. A polynomial in n and m denotes the running time of an algorithm. We left out the O-notation to improve readability. References are usually given following the running time. If not then this means that the algorithm is trivial. We use the abbreviation NPC for NP-complete problems. A question mark indicates that a polynomial time algorithm seems not to be known. A question mark together with a reference indicates that finding a polynomial time algorithm for

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this problem is posed as an open problem in the literature.

class	recognition	ω	χ	α	θ
alternately colorable	P [61]	?	?	?	?
alternately orientable	P [61]	?	?	?	?
AT-free Berge	P [18]	?	?	n^4 [15]	?
BIP*	? [21]	?	?	?	?
bipartite	$n+m$	$n+m$	$n+m$	$\sqrt{n}m$ [67]	P [40]
brittle	m^2 [109]	nm [55]	nm [55]	nm [55]	nm [55]
bull-free Berge	n^5 [99]	P [31]	P [31]	P [31]	P [31]
C_4-free Berge	P [18]	?	?	?	?
chair-free Berge	P [18]	?	?	P [2]	?
claw-free Berge	P [25]	$n^{7/2}$ [72]	n^4 [69]	n^4 [108, 91, 81]	$n^{11/2}$ [72]
clique-separable	P [41, 120]	P [41, 120]	P [41, 120]	P [115, 120]	P [120]
cograph contraction	P [79]	nm [55]	nm [55]	nm [55]	nm [55]
comparability	n^2 [111]	n^2 [111, 45]	n^2 [111, 45]	P [45]	P [45]
$\Delta \leq 6$ Berge	P [18]	P	?	?	?
dart-free Berge	P [23]	?	?	?	?
degenerate Berge	? [1]	?	?	?	?
diamond-free Berge	P [36]	?	n^3 [119]	?	?
doc-free Berge	?	?	?	?	?
elementary	P	$n^{7/2}$ [72]	n^4 [69]	n^4 [108, 91, 81]	$n^{11/2}$ [72]
forest	n	n	n	n	P [40]
gem-free Berge	P [18]	?	?	?	?
HHD-free	n^3 [66]	$n+m$ [74]	$n+m$ [74]	$n+m$ [74]	$n+m$ [74]
Hoàng	P	?	?	?	?
i-triangulated	nm [103]	P [41, 120]	$n+m$ [101]	P [115, 120]	P [120]
interval	$n+m$ [11]	$n+m$ [11]	$n+m$ [11]	$n+m$ [11]	$n+m$ [11]
K_4-free Berge	P [18]	P	?	?	?
(K_5, P_5)-free Berge	P	n^4 [82]	? [82]	? [82]	? [82]
LGBIP	$n+m$ [105]	$n+m$ [105]	$n \log n$ [27]	$\sqrt{n}m$ [67]	$n^{11/2}$ [72]
line perfect	P [116]	?	?	?	?
locally perfect	? [98]	?	?	?	?
Meyniel	m^2 [102]	n^3 [59]	n^2 [104]	?	?
murky	P	?	?	?	?
1-overlap bipartite	P	?	?	?	?

class	recognition	ω	χ	α	θ
opposition	?	?	?	?	?
P_4-free	$n+m$ [30]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]
P_4-lite	P	$n+m$ [42]	$n+m$ [42]	$n+m$ [42]	$n+m$ [42]
P_4-reducible	P [75]	$n+m$ [42]	$n+m$ [42]	$n+m$ [42]	$n+m$ [42]
P_4-sparse	$n+m$ [77]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]
P_4-stable Berge	NPC [64]	?	?	?	?
parity	$n+m$ [26]	P [16]	P [16]	P [16]	P [16]
partner-graph △-free	P	?	?	?	?
paw-free Berge	P [18]	n^3 [59]	n^2 [104]	?	?
perfectly contractile	?	?	?	?	?
perfectly orderable	NPC [90]	?	?	?	?
permutation	$n+m$	P [45]	P [45]	P [45]	P [45]
planar Berge	n^3 [70]	$n+m$ [95]	$n^{3/2}$ [71, 113]	P [71]	?
preperfect	?	?	?	?	?
quasi-parity	? [89]	?	?	?	?
Raspail	? [22]	?	?	?	?
skeletal	?	?	?	?	?
slender	?	?	?	?	?
slightly triangulated	P [85]	?	? [85]	?	?
slim	?	?	?	?	?
snap	? [83]	nm [83]	? [83]	?	?
split	$n+m$ [50]	P [45]	P [45]	P [45]	P [45]
strict opposition	?	?	?	?	?
strict quasi-parity	? [89]	?	?	?	?
strongly perfect	?	?	?	?	?
3-overlap bipartite	P [62]	?	?	?	?
3-overlap △-free	P [18]	?	?	?	?
threshold	P	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]
tree	n	n	n	n	P [40]
triangulated	$n+m$ [100]	$n+m$ [100]	$n+m$ [100]	$n+m$ [100]	$n+m$ [100]
trivially perfect	$n+m$ [44]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]	$n+m$ [5]
2-overlap bipartite	P [62]	?	?	?	?
2-overlap △-free	P [18]	?	?	?	?
2-split Berge	P [65]	P [65]	?	P [3]	?
unimodular	?	?	?	?	?
weakly triangulated	n^2m [112]	nm [55]	nm [55]	nm [55]	nm [55]
wing triangulated	P [68]	?	?	?	?

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

5 The Number of Perfect Graphs

We have implemented an algorithm to check whether a given graph is perfect and counted the number of non-isomorphic perfect graphs on up to 12 vertices. Table 1 contains these numbers and compares them to the number of all non-isomorphic graphs on the same number of vertices. Note that these numbers include disconnected graphs. It is well known that the proportion of graphs which are perfect tends to zero (see for example Proposition 11.3.1 in [32]).

Table 1: The number of all non-isomorphic graphs and the number of all non-isomorphic perfect graphs on exactly n vertices for $n = 5, \ldots, 12$.

	5	6	7	8	9	10	11	12
all graphs	34	156	1044	12346	274668	12005168	1018997864	165091172592
perfect	33	148	906	8887	136756	3269264	115811998	5855499195

We also implemented for each of the 120 classes of perfect graphs an algorithm for recognizing these graphs. We ran these 120 algorithms on all graphs with up to 10 vertices. The following table contains the number of graphs contained in each class for a given number of vertices. These numbers give some impression of how large the classes are. Note that we did not include the complements of the classes in the table, as the complement of a class contains the same number of graphs as the class itself.

class	2	3	4	5	6	7	8	9	10
perfect	2	4	11	33	148	906	8887	136756	3269264
alternately colorable	2	4	11	32	136	749	6142	71759	1174550
alternately orientable	2	4	11	33	147	896	8673	130683	3012745
AT-free Berge	2	4	11	33	144	826	6836	76322	1126575
BIP*	2	4	11	33	147	896	8683	131332	3065093
bipartite	2	3	7	13	35	88	303	1119	5479
brittle	2	4	11	33	146	886	8472	125262	2799594
bull-free Berge	2	4	11	32	130	592	3275	19546	126842
C_4-free Berge	2	4	10	27	95	398	2164	14945	131562
chair-free Berge	2	4	11	32	126	546	2766	15014	88460
claw-free Berge	2	4	10	25	80	262	1003	4044	17983
clique-separable	2	4	11	32	129	630	4118	34375	364004
cograph contraction	2	4	11	33	139	737	5220	47299	542268
comparability	2	4	11	33	144	824	6793	75400	1107853

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

class	2	3	4	5	6	7	8	9	10
$\Delta \leq 6$ Berge	2	4	11	33	148	906	7981	84637	922648
dart-free Berge	2	4	11	32	124	512	2495	13245	79734
degenerate Berge	2	4	11	33	148	906	8884	136682	3265152
diamond-free Berge	2	4	10	24	75	249	1033	4918	28077
doc-free Berge	2	4	11	31	122	560	3395	24891	215455
elementary	2	4	10	25	79	253	936	3601	15486
forest	2	3	6	10	20	37	76	153	329
gem-free Berge	2	4	11	32	130	625	3964	30929	297142
HHD-free	2	4	11	32	128	608	3689	27238	244922
Hoàng	2	4	11	33	145	848	7111	77067	1007506
i-triangulated	2	4	11	31	117	504	2772	18738	158931
interval	2	4	10	27	92	369	1807	10344	67659
K_4-free Berge	2	4	10	28	112	568	4184	42450	576926
(K_5, P_5)-free Berge	2	4	11	31	124	565	3162	19531	132566
LGBIP	2	4	9	17	39	84	200	484	1263
line perfect	2	4	11	26	80	248	899	3441	15081
locally perfect	2	4	11	33	148	901	8664	126954	2769696
Meyniel	2	4	11	32	130	622	3839	28614	258660
murky	2	4	11	33	146	850	7069	77493	1072620
1-overlap bipartite	2	4	11	33	148	902	6349	38037	210384
opposition	2	4	11	33	146	848	6880	68743	778449
P_4-free	2	4	10	24	66	180	522	1532	4624
P_4-lite	2	4	11	33	94	278	841	2613	8314
P_4-reducible	2	4	11	27	76	212	631	1893	5846
P_4-sparse	2	4	11	27	78	218	653	1963	6088
P_4-stable Berge	2	4	11	33	147	894	8515	120263	2363930
parity	2	4	11	31	116	466	2207	11258	63098
partner-graph $\triangle$-free	2	4	11	33	132	494	1603	5038	16334
paw-free Berge	2	4	10	21	54	130	395	1323	5946
perfectly contractile	2	4	11	33	147	896	8683	131333	3065118
perfectly orderable	2	4	11	33	147	896	8682	131299	3062755
permutation	2	4	11	33	142	776	5699	50723	524572
planar Berge	2	4	11	32	134	711	5229	48736	543955
preperfect	2	4	11	33	148	906	8887	136755	3269254
quasi-parity	2	4	11	33	148	906	8886	136735	3268600
Raspail	2	4	11	33	148	901	8690	127853	2803340

class	2	3	4	5	6	7	8	9	10
skeletal	2	4	11	33	145	826	6266	54401	504200
slender	2	4	11	33	148	875	7675	93735	1557742
slightly triangulated	2	4	11	33	147	896	8682	131293	3059990
slim	2	4	11	33	147	892	8335	109568	1845372
snap	2	4	11	33	147	896	8677	130114	2951360
split	2	4	9	21	56	164	557	2223	10766
strict opposition	2	4	11	33	145	840	6757	66677	742244
strict quasi-parity	2	4	11	33	147	896	8684	131363	3066504
strongly perfect	2	4	11	33	147	896	8682	131303	3063185
3-overlap bipartite	2	4	11	33	134	492	1634	5127	16624
3-overlap △-free	2	4	11	33	136	532	1783	5549	17906
threshold	2	4	8	16	32	64	128	256	512
tree	1	1	2	3	6	11	23	47	106
triangulated	2	4	10	27	94	393	2119	14524	126758
trivially perfect	2	4	9	20	48	115	286	719	1842
2-overlap bipartite	2	4	11	33	138	582	2367	9421	37916
2-overlap △-free	2	4	11	33	140	586	2379	9495	38436
2-split Berge	2	4	11	33	148	906	8887	136750	3268816
unimodular	2	4	11	33	144	822	6744	73147	1006995
weakly triangulated	2	4	11	33	146	886	8483	126029	2866876
wing triangulated	2	4	11	33	133	598	2836	13304	62243

6 Relations Between Classes of Perfect Graphs

This section contains a table of all known relations between the 120 classes of perfect graphs covered in this paper. The table contains 14400 entries. There exist 150 cases in which the relation between two classes are not known. Several of these undetermined relations are well known open problems. This table contains two entries that have been open problems before. We show that the class of strict quasi-parity graphs is not contained in the class of perfectly contractile graphs as was asked in [10], and we show that (K_5, P_5)-free Berge graphs are not quasi-parity, as was asked in [82].

In the following we list the undetermined relations which have been posed as open problems in the literature and give references.

alternately orientable $\in$ quasi-parity [89, 61]
alternately orientable $\in$ strict quasi-parity [61, 22]
BIP* $\in$ quasi-parity [89]

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

BIP* ∈ strict quasi-parity [22]
1-overlap bipartite ∈ quasi-parity [62]
quasi parity ∈ preperfect [49]
slim ∈ BIP* [56]
slim ∈ strict quasi-parity [56]
slender ∈ quasi-parity [57]
strongly perfect∈ perfectly contractile [10]
strongly perfect ∈ quasi-parity [89]
strongly perfect ∈ strict quasi-parity [22]

The table is split over several pages. Here is a short description on how to use the table. In the upper left corner you find a small map helping you to find out which part of the table you are currently looking at. If you are interested in knowing whether a class $\mathcal{C}_1$ is a subclass of $\mathcal{C}_2$, find the cell in the intersection of the row containing class $\mathcal{C}_1$ and the column containing class $\mathcal{C}_2$. If the cell contains a "=" then the two classes are the same. If the cell contains a "<" or a "$\lessdot$" then $\mathcal{C}_1$ is a proper subclass of $\mathcal{C}_2$. Here, "$\lessdot$" denotes inclusions that belong to the transitive reduction of the inclusion-order. If the cell is empty (gray) then it is not known whether $\mathcal{C}_1$ is a subclass of $\mathcal{C}_2$. In all other cases class $\mathcal{C}_1$ is *not* a subclass of $\mathcal{C}_2$. In this case you will find some letters and numbers in the cell, which describe an example of a graph which is contained in $\mathcal{C}_1$ but not in $\mathcal{C}_2$ and have the following meaning:

K_i	clique of size i
I_i	stable set of size i
P_i	path with i vertices
C_i	cycle with i vertices
$K_{n,m}$	a complete bipartite graph with n respectively m vertices on each side
nG	n disjoint copies of the graph G
$\overline{G}$	the complement of G
F_i	this graph is described in Section 7

Almost all of the counterexamples appearing in this table were found by a computer program by "simply" scanning all 3416012 perfect graphs on up to 10 vertices. For each of these graphs it was checked to which of the 120 classes it belongs. As several of these membership tests require exponential time the total running time was about two month on a 1.3GHz PC.

All counterexamples given in the table which have at most 10 vertices are smallest possible with respect to the number of vertices.

Only the (currently 9) graphs on more than 10 vertices had to be found by hand. Clearly the inclusions cannot be proven by a computer. However, only the transitive

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

reduction of the inclusion-order has to be typed in by hand, the transitive closure of the relations is generated automatically (including consistency checks). Thus in total out of the currently 14400 entries only 237 had to be made by hand.

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Inclusions between classes of perfect graphs

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

	alternately colorable	alternately orientable	AT-free Berge	BIP*	bipartite	brittle	bull-free Berge	C_4-free Berge	chair-free Berge	claw-free Berge	clique-separable	co-alternately colorable	co-alternately orientable	co-AT-free Berge	co-BIP*	co-bipartite	co-chair-free Berge	co-claw-free Berge	co-clique-separable	co-cograph contraction
alternately colorable	$=$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
alternately orientable	$K_{2,3}$	$=$	C_6	$\lessdot$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
AT-free Berge	$K_{2,3}$	$\overline{C_6}$	$=$	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$		$\overline{C_6}$	$\lessdot$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
BIP*	$K_{2,3}$	F_{41}	C_6	$=$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
bipartite	$K_{2,3}$	$<$	C_6	$<$	$=$	C_6	$<$	C_4	$\overline{F_7}$	$K_{1,3}$	$<$	$<$	C_6	$<$	C_6	I_3	$<$	$<$	F_4	P_6
brittle	$K_{2,3}$	F_{41}	F_{15}	$<$	K_3	$=$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\overline{F_{41}}$	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
bull-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	$=$	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
C_4-free Berge	F_{43}	F_{55}	C_6	F_{55}	K_3	C_6	F_5	$=$	$\overline{F_7}$	$K_{1,3}$	F_{29}	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
chair-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$=$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
claw-free Berge	$\overline{F_{54}}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\lessdot$	$=$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
clique-separable	$K_{2,3}$	$\lessdot$	C_6	$<$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$=$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-alternately colorable	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$=$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-alternately orientable	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$=$	$\overline{C_6}$	$\lessdot$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-AT-free Berge	$K_{2,3}$		C_6	$\lessdot$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$=$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-BIP*	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\overline{F_{41}}$	$\overline{C_6}$	$=$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-bipartite	$<$	$\overline{C_6}$	$<$	$\overline{C_6}$	K_3	$\overline{C_6}$	$<$	C_4	$<$	$<$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$\overline{C_6}$	$<$	$=$	F_7	$\overline{K_{1,3}}$	$<$	$\overline{C_6}$
co-chair-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	$=$	$\overline{K_{1,3}}$	F_4	P_6
co-claw-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	F_{54}	C_6	$\overline{C_6}$	C_6	I_3	$\lessdot$	$=$	F_4	P_6
co-clique-separable	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\lessdot$	$\overline{C_6}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	$=$	$\overline{C_6}$
co-cograph contraction	$K_{2,3}$	F_{56}	F_{15}	$<$	K_3	F_{42}	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{39}	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	$=$
co-comparability	$K_{2,3}$	$\overline{C_6}$	$\lessdot$	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\lessdot$	$\overline{C_6}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-$\Delta \leq 6$ Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-dart-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-degenerate Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-diamond-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\lessdot$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	C_6	C_6
co-doc-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	C_6
co-elementary	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\lessdot$	C_6	$\overline{C_6}$	C_6	I_3	$<$	$\lessdot$	F_4	P_6
co-forest	$<$	$\overline{F_{61}}$	$<$	$<$	K_3	$<$	$<$	C_4	$<$	$<$	$\overline{F_4}$	$<$	$<$	$\overline{F_{24}}$	$<$	$\lessdot$	F_7	$\overline{K_{1,3}}$	$<$	$\overline{P_7}$
co-gem-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	C_6
co-HHD-free	$K_{2,3}$	F_{56}	F_{15}	$<$	K_3	$\lessdot$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\overline{F_{56}}$	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	$\overline{F_{11}}$

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Inclusions between classes of perfect graphs

1	**2**	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

	co-comparability	co-$\Delta \leq 6$ Berge	co-dart-free Berge	co-degenerate Berge	co-diamond-free Berge	co-doc-free Berge	co-elementary	co-forest	co-gem-free Berge	co-HHD-free	co-Hoàng	co-i-triangulated	co-interval	co-(K_5, P_5)-free Berge	co-LGBIP	co-line perfect	co-locally perfect	co-Meyniel	co-opposition	co-P_4-stable Berge
alternately colorable	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
alternately orientable	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
AT-free Berge	F_{21}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{23}
BIP*	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
bipartite	C_6	I_8	$<$	$4K_2$	F_1	F_3	$\lessdot$	I_3	F_3	P_5	$<$	P_5	$\overline{C_4}$	I_5	F_1	I_5	P_7	P_5	F_{24}	C_6
brittle	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	F_{27}
bull-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
C_4-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
chair-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
claw-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
clique-separable	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
co-alternately colorable	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-alternately orientable	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-AT-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{14}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
co-BIP*	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-bipartite	$<$	$\overline{K_{1,7}}$	$<$	$\overline{K_{4,4}}$	$\lessdot$	$<$	$\overline{K_{1,3}}$	$\overline{C_4}$	$<$	$\overline{C_6}$	$\overline{C_6}$	$<$	$\overline{C_4}$	$\overline{P_5}$	$\overline{K_{1,3}}$	$\lessdot$	$<$	$<$	$\overline{C_6}$	$<$
co-chair-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-claw-free Berge	C_6	I_8	$\lessdot$	$4K_2$	F_1	F_3	F_{15}	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	F_1	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-clique-separable	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$\lessdot$	P_5	$\overline{C_6}$	F_{27}
co-cograph contraction	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{22}	P_5	$\overline{F_{14}}$	F_{27}
co-comparability	$=$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{23}
co-$\Delta \leq 6$ Berge	C_6	$=$	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-dart-free Berge	C_6	I_8	$=$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-degenerate Berge	C_6	I_8	$\overline{F_6}$	$=$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-diamond-free Berge	C_6	I_8	$\lessdot$	$\overline{K_{4,4}}$	$=$	$\lessdot$	$\overline{K_{1,3}}$	I_3	$<$	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{57}	P_5	$\overline{C_6}$	C_6
co-doc-free Berge	C_6	I_8	$\overline{F_6}$	$\overline{K_{4,4}}$	F_1	$=$	$\overline{K_{1,3}}$	I_3	$\lessdot$	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{22}	P_5	$\overline{C_6}$	C_6
co-elementary	C_6	I_8	$<$	$4K_2$	F_1	F_3	$=$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	F_1	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-forest	$<$	$\overline{K_{1,7}}$	$<$	$<$	$<$	$<$	$\overline{K_{1,3}}$	$=$	$<$	$<$	$\overline{F_{35}}$	$<$	$\overline{F_{24}}$	$\overline{P_5}$	$\overline{K_{1,3}}$	$<$	$<$	$<$	$\overline{F_{35}}$	$<$
co-gem-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	$\overline{F_8}$	$\overline{K_{1,3}}$	I_3	$=$	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{22}	P_5	$\overline{C_6}$	C_6
co-HHD-free	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$=$	$\overline{F_{15}}$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$\lessdot$	$\overline{F_{14}}$	F_{27}

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Inclusions between classes of perfect graphs

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Inclusions between classes of perfect graphs	co-parity	co-paw-free Berge	co-perfectly contractile	co-perfectly orderable	co-planar Berge	co-Raspail	co-skeletal	co-slender	co-slightly triangulated	co-slim	co-snap	co-strict opposition	co-strict quasi-parity	co-strongly perfect	co-tree	co-triangulated	co-trivially perfect	co-unimodular	co-wing triangulated	cograph contraction
alternately colorable	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
alternately orientable	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
AT-free Berge	P_5	F_2			I_5	$\overline{F_{51}}$	F_{16}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$			K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
BIP*	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
bipartite	P_5	F_2	C_6	C_6	I_5	$<$	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
brittle	P_5	F_2	$<$	$\lessdot$	I_5	F_{31}	F_{15}	F_{19}	$2P_4$	$\overline{F_{31}}$	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
bull-free Berge	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
C_4-free Berge	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
chair-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
claw-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{29}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
clique-separable	P_5	F_2	C_6	C_6	I_5	F_{26}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-alternately colorable	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-alternately orientable	P_5	F_2		F_{42}	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$		F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-AT-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-BIP*	P_5	F_2		F_{42}	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$		F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-bipartite	$<$	$\lessdot$	$<$	$<$	$\overline{K_{3,3}}$	$<$	$<$	$<$	$\overline{C_6}$	$<$	$<$	$\overline{C_6}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$\lessdot$	$\overline{F_{10}}$	$\overline{C_6}$
co-chair-free Berge	P_5	F_2	C_6	C_6	I_5	F_{26}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-claw-free Berge	P_5	F_2	C_6	C_6	I_5	F_{54}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-clique-separable	P_5	F_2	$\lessdot$	F_{42}	I_5	F_{31}	F_{15}	$\overline{F_{26}}$	$\overline{C_6}$	F_{24}	$3K_2$	$\overline{C_6}$	$<$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-cograph contraction	P_5	F_2	$<$	F_{42}	I_5	F_{31}	F_{15}	F_{19}	$2P_4$	F_{24}	$3K_2$	$\overline{F_{14}}$	$<$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
co-comparability	P_5	F_2	$<$	$\lessdot$	I_5	$\lessdot$	F_{16}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-$\Delta \leq 6$ Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-dart-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-degenerate Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-diamond-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	$\overline{F_{26}}$	$\overline{C_6}$	C_6	F_{60}	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$\overline{F_{54}}$	C_6	C_6
co-doc-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$2C_4$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$\overline{F_{40}}$	C_6	C_6
co-elementary	P_5	F_2	C_6	C_6	I_5	$\lessdot$	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-forest	$<$	$<$	$<$	$<$	$\lessdot$	$<$	$<$	$<$	$<$	$<$	$<$	$\overline{F_{35}}$	$<$	$<$	K_2	$\lessdot$	P_4	$<$	$\overline{F_{10}}$	$\overline{P_6}$
co-gem-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-HHD-free	F_3	F_2	$<$	$<$	I_5	$\overline{F_{56}}$	F_{15}	F_{19}	$2P_4$	$<$	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$

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13	14	15	16	17	18
19	20	21	22	23	24

	comparability	$\Delta \leq 6$ Berge	dart-free Berge	degenerate Berge	diamond-free Berge	doc-free Berge	elementary	forest	gem-free Berge	HHD-free	Hoàng	i-triangulated	I_4-free Berge	interval	K_4-free Berge	(K_5, P_5)-free Berge	LGBIP	line perfect	locally perfect	Meyniel
alternately colorable	$\overline{C_6}$	$K_{1,7}$	F_6	$\overline{4K_2}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
alternately orientable	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
AT-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{14}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
BIP*	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
bipartite	$<$	$K_{1,7}$	$<$	$K_{4,4}$	$\lessdot$	$<$	$K_{1,3}$	C_4	$<$	C_6	C_6	$<$	I_4	C_4	$\lessdot$	P_5	$K_{1,3}$	$\lessdot$	$<$	$<$
brittle	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
bull-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
C_4-free Berge	F_{15}	$K_{1,7}$	F_6	F_{72}	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	C_6	C_6	F_{29}	I_4	C_6	K_4	P_5	$K_{1,3}$	K_5	$\overline{F_{65}}$	F_{26}
chair-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
claw-free Berge	$\overline{C_6}$	K_8	$\lessdot$	$\overline{4K_2}$	$\overline{F_1}$	$\overline{F_3}$	$\overline{F_{15}}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$\overline{F_1}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
clique-separable	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\lessdot$	$\overline{P_5}$
co-alternately colorable	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-alternately orientable	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-AT-free Berge	$\overline{F_{21}}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-BIP*	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-bipartite	$\overline{C_6}$	K_8	$<$	$\overline{4K_2}$	$\overline{F_1}$	$\overline{F_3}$	$\lessdot$	K_3	$\overline{F_3}$	$\overline{P_5}$	$<$	$\overline{P_5}$	$\lessdot$	C_4	K_4	K_5	$\overline{F_1}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-chair-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-claw-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-clique-separable	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-cograph contraction	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	F_{22}	$\overline{P_5}$
co-comparability	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{14}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-$\Delta \leq 6$ Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-dart-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-degenerate Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-diamond-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-doc-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-elementary	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-forest	$\overline{F_{24}}$	K_8	$<$	$\overline{4K_2}$	$\overline{F_1}$	$\overline{F_3}$	$<$	K_3	$\overline{F_3}$	$\overline{P_5}$	$<$	$\overline{P_5}$	$<$	C_4	K_4	K_5	$\overline{F_1}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-gem-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-HHD-free	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$

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Inclusions between classes of perfect graphs

1	2	3	4	**5**	6
7	8	9	10	11	12
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	murky	1-overlap bipartite	opposition	P_4-free	P_4-lite	P_4-reducible	P_4-sparse	P_4-stable Berge	parity	partner-graph $\triangle$-free	paw-free Berge	perfectly contractile	perfectly orderable	permutation	planar Berge	preperfect	quasi-parity	Raspail	skeletal	slender
alternately colorable	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
alternately orientable	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$		$\overline{F_{42}}$	C_6	K_5			$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
AT-free Berge	P_6	$\overline{F_{31}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
BIP*	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$		$\overline{F_{42}}$	C_6	K_5			$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
bipartite	P_6	F_{34}	C_6	P_4	P_6	P_5	P_5	$<$	$<$	C_6	$\lessdot$	$<$	$<$	C_6	$K_{3,3}$	$<$	$<$	$<$	$<$	$<$
brittle	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$\lessdot$	F_{15}	K_5	$\lessdot$	$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
bull-free Berge	P_6	F_{34}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5		$\lessdot$	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
C_4-free Berge	P_6	$\overline{F_{27}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	C_6	$\overline{F_2}$	F_{62}	F_{55}	C_6	K_5	F_{74}	F_{68}	$\overline{F_{66}}$	$\overline{F_{15}}$	$\overline{F_{29}}$
chair-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{26}}$	$\overline{C_6}$	$\overline{F_{19}}$
claw-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{54}}$	$\overline{C_6}$	$\overline{F_{19}}$
clique-separable	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\lessdot$	$\overline{F_{42}}$	C_6	K_5	$\lessdot$	$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{26}}$
co-alternately colorable	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-alternately orientable	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-AT-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{23}}$	$\overline{P_5}$	C_6	$\overline{F_2}$			C_6	K_5			$\overline{F_{51}}$	$\overline{F_{16}}$	$\overline{F_{19}}$
co-BIP*	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-bipartite	$\overline{P_6}$	$\overline{F_{34}}$	$\overline{F_{24}}$	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$<$	$<$	$<$	$\overline{C_6}$	$\overline{F_{19}}$
co-chair-free Berge	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-claw-free Berge	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{29}}$
co-clique-separable	$\overline{P_6}$	F_{31}	F_{14}	P_4	$\overline{C_6}$	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$\lessdot$	$<$	$\overline{F_{26}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-cograph contraction	$\overline{P_6}$	F_{31}	F_{14}	P_4	$\overline{P_6}$	P_5	P_5	$\lessdot$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$\lessdot$	F_{15}	K_5		$<$	$\overline{F_{56}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
co-comparability	P_6	$\overline{F_{31}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$<$	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-$\Delta \leq 6$ Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-dart-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-degenerate Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-diamond-free Berge	$\overline{P_6}$	F_{31}	C_6	P_4	C_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{26}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-doc-free Berge	$\overline{P_6}$	F_{31}	C_6	P_4	C_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-elementary	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{29}}$
co-forest	$\overline{P_6}$	$\overline{F_{47}}$	$\overline{F_{24}}$	P_4	$\overline{P_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{2P_4}$	$\overline{P_5}$	$\overline{F_9}$	$\overline{F_2}$	$<$	$<$	$\overline{F_{24}}$	K_5	$<$	$<$	$<$	$\overline{P_8}$	$\overline{F_{19}}$
co-gem-free Berge	$\overline{P_6}$	F_{31}	C_6	P_4	C_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-HHD-free	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{P_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{F_{23}}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$	$\overline{F_{56}}$	$\overline{F_{15}}$	$\overline{F_{19}}$

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13	14	15	16	17	18
19	20	21	22	23	24

	slightly triangulated	slim	snap	split	strict opposition	strict quasi-parity	strongly perfect	3-overlap bipartite	3-overlap $\triangle$-free	threshold	tree	triangulated	trivially perfect	2-overlap bipartite	2-overlap $\triangle$-free	2-split Berge	$2K_2$-free Berge	unimodular	weakly triangulated	wing triangulated
alternately colorable	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
alternately orientable	C_6	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	C_6		$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
AT-free Berge	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
BIP*	C_6	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	C_6		$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
bipartite	C_6	$<$	$<$	$\overline{C_4}$	C_6	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$\lessdot$	$\overline{C_4}$	$\lessdot$	C_6	F_{10}
brittle	$\overline{2P_4}$	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\lessdot$	F_{10}
bull-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
C_4-free Berge	C_6	F_{55}	F_{69}	$\overline{C_4}$	C_6	F_{68}	F_{62}	F_9	F_9	$\overline{C_4}$	I_2	C_6	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{F_{15}}$	C_6	F_{10}
chair-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
claw-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
clique-separable	C_6	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
co-alternately colorable	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-alternately orientable	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-AT-free Berge	C_6	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	C_6			F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
co-BIP*	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-bipartite	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$\lessdot$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-chair-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-claw-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-clique-separable	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-cograph contraction		F_{31}	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\lessdot$	F_{10}
co-comparability	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-$\Delta \leq 6$ Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-dart-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-degenerate Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-diamond-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-doc-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-elementary	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-forest	$\overline{2P_4}$	$\overline{F_{24}}$	$\overline{3K_2}$	C_4	$\overline{F_{24}}$	$<$	$<$	$\overline{F_9}$	$\overline{F_9}$	P_4	I_2	C_4	P_4	$\overline{F_{24}}$	$\overline{F_{24}}$	$<$	$<$	$\overline{3K_2}$	$<$	$\overline{F_{10}}$
co-gem-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-HHD-free	$\overline{2P_4}$	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$<$	$\overline{F_{10}}$

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	alternately colorable	alternately orientable	AT-free Berge	BIP*	bipartite	brittle	bull-free Berge	C_4-free Berge	chair-free Berge	claw-free Berge	clique-separable	co-alternately colorable	co-alternately orientable	co-AT-free Berge	co-BIP*	co-bipartite	co-chair-free Berge	co-claw-free Berge	co-clique-separable	co-cograph contraction
co-Hoàng	$K_{2,3}$	$\overline{C_8}$	C_6	$\overline{C_8}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-i-triangulated	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$\overline{C_6}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	$\lessdot$	$\overline{C_6}$
co-interval	$K_{2,3}$	$<$	F_{27}	$<$	K_3	$<$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$<$	$<$	$<$	$<$	I_3	F_7	$\overline{K_{1,3}}$	$<$	$\overline{P_7}$
co-(K_5, P_5)-free Berge	$K_{2,3}$	F_{56}	C_6	F_{55}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-LGBIP	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$<$	$K_{1,3}$	$\overline{C_6}$	$<$	C_6	$\overline{C_6}$	C_6	I_3	$<$	$<$	C_6	C_6
co-line perfect	$K_{2,3}$	$\overline{C_6}$	$\overline{F_{15}}$	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$\overline{C_6}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	$<$	$\overline{C_6}$
co-locally perfect	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-Meyniel	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{56}	$\overline{C_6}$	$\lessdot$	I_3	F_7	$\overline{K_{1,3}}$	F_4	$\overline{C_6}$
co-opposition	$K_{2,3}$	$\overline{C_8}$	C_6	$\overline{C_8}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-P_4-stable Berge	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$	F_{44}	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-parity	$K_{2,3}$	$\overline{C_6}$	$\overline{F_{15}}$	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{56}	$\overline{C_6}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	$\overline{C_6}$
co-paw-free Berge	$K_{2,3}$	$\overline{C_6}$	$<$	$\overline{C_6}$	K_3	$\overline{C_6}$	$\lessdot$	C_4	$\lessdot$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$\overline{C_6}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	$<$	$\overline{C_6}$
co-perfectly contractile	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$	F_{55}	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-perfectly orderable	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$	$\lessdot$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-planar Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-Raspail	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-skeletal	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$		I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-slender	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-slightly triangulated	$K_{2,3}$	F_{41}	C_6	F_{44}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-slim	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$		I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-snap	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-strict opposition	$K_{2,3}$	$\overline{C_8}$	F_{15}	$\overline{C_8}$	K_3	$\overline{C_8}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	F_{15}	$\lessdot$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-strict quasi-parity	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$	F_{44}	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-strongly perfect	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$	F_{55}	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-tree	$<$	$\overline{F_{61}}$	$<$	$<$	$\overline{K_{1,3}}$	$<$	$<$	$\overline{P_5}$	$<$	$<$	$\overline{P_6}$	$<$	$<$	$\overline{F_{24}}$	$<$	$<$	F_7	$\overline{K_{1,3}}$	$<$	$\overline{P_7}$
co-triangulated	$K_{2,3}$	F_{56}	F_{15}	$<$	K_3	$<$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\lessdot$	$<$	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	$<$	$\overline{P_7}$
co-trivially perfect	$K_{2,3}$	$<$	$<$	$<$	K_3	$<$	$<$	C_4	$<$	$K_{1,3}$	$\overline{F_4}$	$<$	$<$	$<$	$<$	I_3	$<$	$\overline{K_{1,3}}$	$<$	$<$
co-unimodular	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
co-wing triangulated	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{45}	$\overline{C_6}$	F_{55}	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
cograph contraction	$K_{2,3}$	$\overline{F_{39}}$	F_{15}	$<$	K_3	$\overline{F_{42}}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{56}	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6

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19	20	21	22	23	24

	co-comparability	co-$\Delta \leq 6$ Berge	co-dart-free Berge	co-degenerate Berge	co-diamond-free Berge	co-doc-free Berge	co-elementary	co-forest	co-gem-free Berge	co-HHD-free	co-Hoàng	co-i-triangulated	co-interval	co-(K_5, P_5)-free Berge	co-LGBIP	co-line perfect	co-locally perfect	co-Meyniel	co-opposition	co-P_4-stable Berge
co-Hoàng	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$=$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{24}}$	C_6
co-i-triangulated	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$\overline{C_6}$	$\overline{C_6}$	$=$	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$\lessdot$	$\overline{C_6}$	F_{27}
co-interval	F_{27}	I_8	$\overline{F_6}$	$<$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$<$	$\overline{F_{35}}$	$<$	$=$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$\overline{F_{35}}$	F_{27}
co-(K_5, P_5)-free Berge	C_6	$\overline{K_{1,7}}$	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	$=$	$\overline{K_{1,3}}$	P_5	P_7	P_5	$\overline{F_{14}}$	C_6
co-LGBIP	C_6	I_8	$<$	$\overline{F_{71}}$	$\lessdot$	$<$	$\lessdot$	I_3	$<$	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$=$	I_5	F_{57}	P_5	$\overline{C_6}$	C_6
co-line perfect	$\overline{F_{15}}$	$\overline{K_{1,7}}$	$\overline{F_6}$	$\overline{K_{4,4}}$	F_1	$\lessdot$	$\overline{K_{1,3}}$	I_3	$<$	$\overline{C_6}$	$\overline{C_6}$	$\lessdot$	$\overline{C_4}$	$\overline{P_5}$	$\overline{K_{1,3}}$	$=$	$<$	$<$	$\overline{C_6}$	F_{58}
co-locally perfect	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$=$	P_5	$\overline{C_6}$	C_6
co-Meyniel	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$\overline{C_6}$	$\overline{C_6}$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$\lessdot$	$=$	$\overline{C_6}$	F_{27}
co-opposition	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$=$	C_6
co-P_4-stable Berge	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	$=$
co-parity	$\overline{F_{15}}$	I_8	$\overline{F_6}$	$4K_2$	F_1	$\overline{F_8}$	$\overline{K_{1,3}}$	I_3	$\lessdot$	$\overline{C_6}$	$\overline{C_6}$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$\lessdot$	$\overline{C_6}$	F_{36}
co-paw-free Berge	$\lessdot$	I_8	$\lessdot$	$4K_2$	F_1	$\overline{F_8}$	$\overline{K_{1,3}}$	I_3	$<$	$\overline{C_6}$	$\overline{C_6}$	$\lessdot$	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$\overline{C_6}$	$\lessdot$
co-perfectly contractile	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-perfectly orderable	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-planar Berge	C_6	$\overline{K_{1,7}}$	$\overline{F_6}$		F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	$\overline{P_5}$	$\overline{K_{1,3}}$	P_5		P_5	$\overline{C_6}$	C_6
co-Raspail	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-skeletal	$\overline{F_{15}}$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{34}
co-slender	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-slightly triangulated	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
co-slim	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-snap	C_6	I_8	$\overline{F_6}$	$\overline{K_{4,4}}$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
co-strict opposition	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\lessdot$	F_{27}
co-strict quasi-parity	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-strongly perfect	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
co-tree	$<$	$\overline{K_{1,7}}$	$<$	$<$	$<$	$<$	$\overline{K_{1,3}}$	$\lessdot$	$<$	$<$	$\overline{F_{35}}$	$<$	$\overline{F_{24}}$	$\overline{P_5}$	$\overline{K_{1,3}}$	$<$	$<$	$<$	$\overline{F_{35}}$	$<$
co-triangulated	F_{15}	I_8	$\overline{F_6}$	$\lessdot$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$\lessdot$	$\overline{F_{15}}$	$\lessdot$	F_{15}	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$\overline{F_{35}}$	F_{27}
co-trivially perfect	$<$	I_8	$\overline{F_6}$	$<$	F_1	$\lessdot$	$\overline{K_{1,3}}$	I_3	$<$	$<$	$<$	$<$	$\lessdot$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$<$	$<$
co-unimodular	C_6	I_8	$\overline{F_6}$	$\overline{K_{4,4}}$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{22}	P_5	$\overline{C_6}$	C_6
co-wing triangulated	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	$2P_4$
cograph contraction	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{22}	P_5	$\overline{F_{14}}$	$\lessdot$

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Inclusions between classes of perfect graphs

1	2	3	4	5	6
7	8	**9**	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

	co-parity	co-paw-free Berge	co-perfectly contractile	co-perfectly orderable	co-planar Berge	co-Raspail	co-skeletal	co-slender	co-slightly triangulated	co-slim	co-snap	co-strict opposition	co-strict quasi-parity	co-strongly perfect	co-tree	co-triangulated	co-trivially perfect	co-unimodular	co-wing triangulated	cograph contraction
co-Hoàng	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-i-triangulated	F_3	F_2	$<$	$\overline{F_{73}}$	I_5	$\lessdot$	F_{15}	$\lessdot$	$\overline{C_6}$	$<$	$3K_2$	$\overline{C_6}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-interval	F_3	F_2	$<$	$<$	I_5	$<$	F_{27}	$<$	$<$	$<$	$<$	$\overline{F_{35}}$	$<$	$<$	K_2	$\lessdot$	P_4	$\lessdot$	F_{10}	$\overline{P_6}$
co-(K_5, P_5)-free Berge	P_5	F_2	C_6	C_6	$2P_3$	F_{29}	C_6	F_{29}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-LGBIP	P_5	F_2	C_6	C_6	I_5	$<$	C_6	$\overline{F_{26}}$	$\overline{C_6}$	C_6	F_{60}	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$\lessdot$	C_6	C_6
co-line perfect	$\lessdot$	F_2	$<$	$\overline{F_{73}}$	$\overline{K_{3,3}}$	$<$	$<$	$<$	$\overline{C_6}$	$<$	$\lessdot$	$\overline{C_6}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$		$\overline{F_{10}}$	$\overline{C_6}$
co-locally perfect	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-Meyniel	F_3	F_2	$\lessdot$	$\overline{F_{67}}$	I_5	$\overline{F_{56}}$	F_{15}	F_{19}	$\overline{C_6}$	$\lessdot$	$3K_2$	$\overline{C_6}$	$<$	$\lessdot$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-opposition	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-P_4-stable Berge	P_5	F_2	$\overline{F_{44}}$	F_{42}	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$	$\overline{F_{59}}$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-parity	$=$	F_2	$<$	$\overline{F_{73}}$	I_5	$\overline{F_{56}}$	$\lessdot$	F_{19}	$\overline{C_6}$	$<$	$3K_2$	$\overline{C_6}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-paw-free Berge	$\lessdot$	$=$	$<$	$<$	I_5	$<$	$<$	$<$	$\overline{C_6}$	$<$	$3K_2$	$\overline{C_6}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	$\overline{F_{10}}$	$\overline{C_6}$
co-perfectly contractile	P_5	F_2	$=$	F_{42}	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$	$\lessdot$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-perfectly orderable	P_5	F_2	$\lessdot$	$=$	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$	$<$	$\lessdot$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-planar Berge	P_5	F_2	C_6	C_6	$=$	F_{31}	C_6	$\overline{F_{29}}$	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-Raspail	P_5	F_2	C_6	C_6	I_5	$=$	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-skeletal	P_5	F_2		$\overline{F_{63}}$	I_5	$\overline{F_{28}}$	$=$	F_{19}	$\overline{C_6}$	F_{24}	$3K_2$	$\overline{C_6}$	$\lessdot$	$\overline{F_{63}}$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-slender	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	$=$	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-slightly triangulated	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$=$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
co-slim	P_5	F_2		$\overline{F_{54}}$	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$=$	$3K_2$	$\overline{C_6}$		$\overline{F_{54}}$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-snap	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$=$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	F_{15}	C_6	C_6
co-strict opposition	P_5	F_2			I_5	$\overline{F_{28}}$	F_{15}	F_{19}	$2P_4$	$\overline{F_{31}}$	$3K_2$	$=$	$\lessdot$		K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
co-strict quasi-parity	P_5	F_2	$\overline{F_{44}}$	F_{42}	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$	$=$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-strongly perfect	P_5	F_2		$\overline{F_{55}}$	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$		$=$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
co-tree	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$\overline{F_{35}}$	$<$	$<$	$=$	$<$	P_4	$<$	$\overline{F_{10}}$	$\overline{P_6}$
co-triangulated	F_3	F_2	$<$	$<$	I_5	$<$	F_{15}	$<$	$\lessdot$	$<$	$\lessdot$	$\overline{F_{35}}$	$<$	$<$	K_2	$=$	P_4	F_{15}	F_{10}	$\overline{P_6}$
co-trivially perfect	$<$	F_2	$<$	$<$	I_5	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	K_2	$<$	$=$	$<$	$<$	$<$
co-unimodular	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$2C_4$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$=$	C_6	C_6
co-wing triangulated	P_5	F_2		$\overline{F_{55}}$	I_5	$\overline{F_{53}}$	F_{15}	F_{19}	$\overline{C_6}$	F_{32}	$3K_2$	$\overline{C_6}$	$\lessdot$		K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	$=$	$\overline{C_6}$
cograph contraction	P_5	F_2	$<$	$\lessdot$	I_5	$\overline{F_{56}}$	F_{15}	F_{19}		$\overline{F_{31}}$	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$=$

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	comparability	$\Delta \leq 6$ Berge	dart-free Berge	degenerate Berge	diamond-free Berge	doc-free Berge	elementary	forest	gem-free Berge	HHD-free	Hoàng	i-triangulated	I_4-free Berge	interval	K_4-free Berge	(K_5, P_5)-free Berge	LGBIP	line perfect	locally perfect	Meyniel
co-Hoàng	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-i-triangulated	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-interval	$\lessdot$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{14}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-(K_5, P_5)-free Berge	F_{15}	K_8	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	C_6	C_6	$\overline{F_4}$	I_4	C_4	K_4	P_5	$K_{1,3}$	K_5	$\overline{F_{65}}$	F_{26}
co-LGBIP	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-line perfect	$\overline{C_6}$	K_8	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{49}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-locally perfect	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-Meyniel	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-opposition	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-P_4-stable Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-parity	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{49}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-paw-free Berge	$\overline{C_6}$	$K_{1,7}$	$\lessdot$	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	$\lessdot$	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-perfectly contractile	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-perfectly orderable	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-planar Berge	$\overline{C_6}$	K_8	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-Raspail	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-skeletal	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{14}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-slender	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-slightly triangulated	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-slim	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-snap	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-strict opposition	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-strict quasi-parity	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-strongly perfect	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-tree	$\overline{F_{24}}$	$\overline{P_9}$	$<$	$\overline{F_{48}}$	F_7	$\overline{P_6}$	$<$	$\overline{K_{1,3}}$	$\overline{P_6}$	$\overline{P_5}$	$<$	$\overline{P_5}$	$<$	$\overline{P_5}$	$\overline{K_{1,4}}$	$\overline{K_{1,5}}$	F_7	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-triangulated	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-trivially perfect	$<$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	F_8	$K_{1,3}$	K_3	$<$	$<$	$<$	$\overline{F_4}$	I_4	C_4	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
co-unimodular	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
co-wing triangulated	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
cograph contraction	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	F_{22}	$\overline{P_5}$

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Inclusions between classes of perfect graphs

1	2	3	4	5	6
7	8	9	10	**11**	12
13	14	15	16	17	18
19	20	21	22	23	24

	murky	1-overlap bipartite	opposition	P_4-free	P_4-lite	P_4-reducible	P_4-sparse	P_4-stable Berge	parity	partner-graph $\triangle$-free	paw-free Berge	perfectly contractile	perfectly orderable	permutation	planar Berge	preperfect	quasi-parity	Raspail	skeletal	slender
co-Hoàng	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{23}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_8}$	$\overline{C_8}$	C_6	K_5			$\overline{F_{31}}$	$\overline{F_{16}}$	$\overline{F_{19}}$
co-i-triangulated	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$<$	$<$	F_{56}	$\overline{C_6}$	$\overline{F_{19}}$
co-interval	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{P_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{2P_4}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$<$	F_{27}	K_5	$<$	$<$	$<$	$\overline{F_{25}}$	$\overline{F_{19}}$
co-(K_5, P_5)-free Berge	P_6	$\overline{F_{27}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	C_6	$\overline{F_2}$	F_{62}	F_{55}	C_6	K_5		F_{68}	F_{56}	$\overline{F_{15}}$	$\overline{F_{19}}$
co-LGBIP	$\overline{P_6}$	F_{46}	C_6	P_4	C_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{26}}$	$\overline{C_6}$	F_{30}
co-line perfect	$\overline{P_6}$	$\overline{F_{34}}$	$\overline{F_{24}}$	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$<$	$<$	$\lessdot$	$\overline{C_6}$	$\overline{F_{19}}$
co-locally perfect	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-Meyniel	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$<$	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-opposition	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{23}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_8}$	$\overline{C_8}$	C_6	K_5	F_{74}	$\overline{F_{68}}$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
co-P_4-stable Berge	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$\overline{F_{59}}$	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-parity	$\overline{P_6}$	$\overline{F_{34}}$	$\overline{F_{24}}$	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$\lessdot$	$<$	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-paw-free Berge	$\overline{P_6}$	$\overline{F_{34}}$	$\overline{F_{24}}$	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$<$	$<$		$\overline{C_6}$	$\overline{F_{19}}$
co-perfectly contractile	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$<$	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-perfectly orderable	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$<$	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-planar Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	$\overline{F_{70}}$	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-Raspail	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-skeletal	P_6	$\overline{F_{27}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$<$	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-slender	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-slightly triangulated	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	F_{44}	$\overline{F_{42}}$	C_6	K_5			$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
co-slim	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-snap	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5		F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-strict opposition	P_6	$\overline{F_{31}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{23}}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$\overline{C_8}$	$\overline{C_8}$	F_{15}	K_5		$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
co-strict quasi-parity	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$\lessdot$	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-strongly perfect	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-tree	$\overline{P_6}$	$\overline{F_{47}}$	$\overline{F_{24}}$	P_4	$\overline{P_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{P_9}$	$\overline{P_5}$	$\overline{F_9}$	$\overline{P_5}$	$<$	$<$	$\overline{F_{24}}$	$\overline{K_{1,5}}$	$<$	$<$	$<$	$\overline{P_8}$	$\overline{F_{20}}$
co-triangulated	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{P_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{2P_4}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$	F_{56}	$\overline{F_{15}}$	$\overline{F_{19}}$
co-trivially perfect	$<$	$<$	$<$	$\lessdot$	$<$	$<$	$<$	$<$	$<$	$<$	$\overline{F_2}$	$<$	$<$	$<$	K_5	$<$	$<$	$<$	$<$	$\overline{F_{19}}$
co-unimodular	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
co-wing triangulated	P_6	$\overline{F_{37}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5		$<$	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
cograph contraction	P_6	$\overline{F_{31}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$\overline{F_{42}}$	F_{15}	K_5		$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$

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	slightly triangulated	slim	snap	split	strict opposition	strict quasi-parity	strongly perfect	3-overlap bipartite	3-overlap $\triangle$-free	threshold	tree	triangulated	trivially perfect	2-overlap bipartite	2-overlap $\triangle$-free	2-split Berge	$2K_2$-free Berge	unimodular	weakly triangulated	wing triangulated
co-Hoàng	C_6	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_8}$	$\overline{C_8}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
co-i-triangulated	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-interval	$\overline{2P_4}$	$\overline{F_{37}}$	$\overline{3K_2}$	C_4	F_{14}	$<$	$<$	F_9	F_9	P_4	I_2	C_4	P_4	F_{14}	F_{14}	$\overline{3K_3}$	$<$	$\overline{3K_2}$	$<$	F_{10}
co-(K_5, P_5)-free Berge	C_6	F_{55}	$\overline{3K_2}$	$\overline{C_4}$	C_6	F_{68}	F_{62}	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
co-LGBIP	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_{12}	F_{12}	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$\overline{3K_3}$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-line perfect	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-locally perfect	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-Meyniel	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-opposition	C_6	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_8}$	$\overline{C_8}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
co-P_4-stable Berge	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-parity	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-paw-free Berge	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-perfectly contractile	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-perfectly orderable	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-planar Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$\overline{3K_3}$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-Raspail	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-skeletal	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-slender	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-slightly triangulated	C_6	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	C_6	F_{64}	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
co-slim	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-snap	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$\overline{3K_3}$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-strict opposition	$\overline{C_8}$	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$\overline{C_8}$	$\overline{C_8}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_8}$	F_{10}
co-strict quasi-parity	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-strongly perfect	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
co-tree	$\overline{P_9}$	$\overline{F_{24}}$	$\overline{F_{24}}$	$\overline{P_5}$	$\overline{F_{24}}$	$<$	$<$	$\overline{F_9}$	$\overline{F_9}$	P_4	I_2	$\overline{P_5}$	P_4	$\overline{F_{24}}$	$\overline{F_{24}}$	$<$	$<$	$\overline{F_{13}}$	$<$	$\overline{F_{10}}$
co-triangulated	$\overline{2P_4}$	$\overline{F_{24}}$	$\overline{3K_2}$	C_4	F_{14}	$<$	$<$	F_9	F_9	P_4	I_2	C_4	P_4	F_{15}	F_{15}	$\overline{3K_3}$	$\lessdot$	$\overline{3K_2}$	$<$	F_{10}
co-trivially perfect	$<$	$<$	$\overline{3K_2}$	C_4	$<$	$<$	$<$	$<$	$<$	C_4	I_2	C_4	C_4	$<$	$<$	$\overline{3K_3}$	$<$	$\overline{3K_2}$	$<$	$<$
co-unimodular	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$\overline{3K_3}$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
co-wing triangulated	$\overline{2P_4}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
cograph contraction	$\overline{2P_4}$	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\lessdot$	F_{10}

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	alternately colorable	alternately orientable	AT-free Berge	BIP*	bipartite	brittle	bull-free Berge	C_4-free Berge	chair-free Berge	claw-free Berge	clique-separable	co-alternately colorable	co-alternately orientable	co-AT-free Berge	co-BIP*	co-bipartite	co-chair-free Berge	co-claw-free Berge	co-clique-separable	co-cograph contraction
comparability	$K_{2,3}$	$\lessdot$	C_6	$<$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\lessdot$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
$\Delta \leq 6$ Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
dart-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
degenerate Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
diamond-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{C_6}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	$\lessdot$	$\overline{K_{1,3}}$	F_4	P_6
doc-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
elementary	$\lessdot$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$<$	$\lessdot$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
forest	$<$	$<$	F_{24}	$<$	$\lessdot$	$<$	$<$	$<$	$\overline{F_7}$	$K_{1,3}$	$<$	$<$	F_{61}	$<$	$<$	I_3	$<$	$<$	F_4	P_6
gem-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
HHD-free	$K_{2,3}$	F_{56}	F_{15}	$<$	K_3	$\lessdot$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\overline{F_{56}}$	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
Hoàng	$K_{2,3}$	$\overline{C_6}$	$\overline{F_{15}}$	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_8	$\overline{C_6}$	C_8	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
i-triangulated	$K_{2,3}$	$<$	C_6	$<$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\lessdot$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
I_4-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
interval	$<$	$<$	$<$	$<$	K_3	$<$	F_5	$<$	$\overline{F_7}$	$K_{1,3}$	$<$	$\overline{K_{2,3}}$	$<$	$\overline{F_{27}}$	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
K_4-free Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
(K_5, P_5)-free Berge	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\overline{F_{56}}$	$\overline{C_6}$	$\overline{F_{55}}$	I_3	F_7	$\overline{K_{1,3}}$	F_4	$\overline{C_6}$
LGBIP	$<$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$<$	$<$	$\overline{C_6}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	$<$	$\overline{K_{1,3}}$	P_6	P_6
line perfect	$K_{2,3}$	$<$	C_6	$<$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$<$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
locally perfect	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
Meyniel	$K_{2,3}$	F_{56}	C_6	$\lessdot$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
murky	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	C_6
1-overlap bipartite	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
opposition	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_8	$\overline{C_6}$	C_8	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
P_4-free	$K_{2,3}$	$<$	$<$	$<$	K_3	$<$	$\lessdot$	C_4	$<$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$<$	$<$	I_3	$<$	$\overline{K_{1,3}}$	F_4	$\lessdot$
P_4-lite	$K_{2,3}$		F_{15}	$<$	K_3	$\lessdot$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$		F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	$\overline{2P_4}$
P_4-reducible	$K_{2,3}$	$<$	$<$	$<$	K_3	$<$	F_5	C_4	$<$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$<$	$<$	I_3	$<$	$\overline{K_{1,3}}$	F_4	$\overline{2P_4}$
P_4-sparse	$K_{2,3}$	$\lessdot$	F_{15}	$<$	K_3	$<$	F_5	C_4	$\lessdot$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\lessdot$	F_{15}	$<$	I_3	$\lessdot$	$\overline{K_{1,3}}$	F_4	$\overline{2P_4}$
P_4-stable Berge	$K_{2,3}$	F_{41}	C_6	F_{44}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
parity	$K_{2,3}$	F_{56}	C_6	$<$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
partner-graph $\triangle$-free	$K_{2,3}$	$\overline{C_8}$	F_{15}	$\overline{C_8}$	K_3	C_8	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_8	F_{15}	C_8	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6

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13	**14**	15	16	17	18
19	20	21	22	23	24

	co-comparability	co-$\Delta \leq 6$ Berge	co-dart-free Berge	co-degenerate Berge	co-diamond-free Berge	co-doc-free Berge	co-elementary	co-forest	co-gem-free Berge	co-HHD-free	co-Hoàng	co-i-triangulated	co-interval	co-(K_5, P_5)-free Berge	co-LGBIP	co-line perfect	co-locally perfect	co-Meyniel	co-opposition	co-P_4-stable Berge
comparability	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{14}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	C_6
$\Delta \leq 6$ Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
dart-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
degenerate Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
diamond-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
doc-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
elementary	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
forest	F_{24}	I_8	$<$	$4K_2$	F_1	F_3	$<$	I_3	F_3	P_5	$<$	P_5	$\overline{C_4}$	I_5	F_1	I_5	P_7	P_5	F_{24}	$2P_4$
gem-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
HHD-free	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{15}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	F_{23}
Hoàng	$\overline{F_{15}}$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{23}
i-triangulated	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{15}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	C_6
I_4-free Berge	C_6	$\overline{K_{1,7}}$	$\overline{F_6}$	$\overline{K_{4,4}}$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	$\overline{P_5}$	$\overline{K_{1,3}}$	P_5	$\lessdot$	P_5	$\overline{C_6}$	C_6
interval	$\lessdot$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{14}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	$2P_4$
K_4-free Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
(K_5, P_5)-free Berge	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$\overline{C_6}$	$\overline{C_6}$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{65}	F_{26}	$\overline{C_6}$	F_{27}
LGBIP	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
line perfect	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{49}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{24}	C_6
locally perfect	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
Meyniel	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{15}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	C_6
murky	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	F_{22}	P_5	$\overline{C_6}$	C_6
1-overlap bipartite	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
opposition	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{23}
P_4-free	$<$	I_8	$\overline{F_6}$	$4K_2$	F_1	$\overline{F_8}$	$\overline{K_{1,3}}$	I_3	$<$	$<$	$<$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$<$	$<$
P_4-lite	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{15}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5		P_5		$2P_4$
P_4-reducible	$<$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$<$	$\lessdot$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$\lessdot$	$2P_4$
P_4-sparse	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$\lessdot$	F_{15}	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$		$2P_4$
P_4-stable Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{15}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	C_6
parity	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{49}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{24}	C_6
partner-graph $\triangle$-free	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{15}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{14}	F_{27}

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	co-parity	co-paw-free Berge	co-perfectly contractile	co-perfectly orderable	co-planar Berge	co-Raspail	co-skeletal	co-slender	co-slightly triangulated	co-slim	co-snap	co-strict opposition	co-strict quasi-parity	co-strongly perfect	co-tree	co-triangulated	co-trivially perfect	co-unimodular	co-wing triangulated	cograph contraction
comparability	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
$\Delta \leq 6$ Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
dart-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
degenerate Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
diamond-free Berge	P_5	F_2	C_6	C_6	I_5	F_{26}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
doc-free Berge	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
elementary	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{29}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
forest	P_5	F_2	$<$	$<$	I_5	$<$	$2P_4$	F_{19}	$2P_4$	F_{24}	$3K_2$	F_{24}	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	P_7
gem-free Berge	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
HHD-free	P_5	F_2	$<$	$<$	I_5	$\overline{F_{56}}$	F_{15}	F_{19}	$2P_4$	F_{24}	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	F_{11}
Hoàng	P_5	F_2	C_8	C_8	I_5	F_{31}	F_{16}	F_{19}	$\overline{C_6}$	F_{24}	$3K_2$	$\overline{C_6}$	C_8	C_8	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
i-triangulated	P_5	F_2	C_6	C_6	I_5	$\overline{F_{56}}$	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
I_4-free Berge	P_5	F_2	C_6	C_6	$\overline{K_{3,3}}$	F_{31}	C_6	F_{29}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
interval	P_5	F_2	$<$	$<$	I_5	$<$	F_{25}	F_{19}	$2P_4$	F_{37}	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	P_7
K_4-free Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
(K_5, P_5)-free Berge	F_3	F_2	$\overline{F_{62}}$	$\overline{F_{55}}$	I_5	$\overline{F_{56}}$	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{55}}$	$3K_2$	$\overline{C_6}$	$\overline{F_{68}}$	$\overline{F_{62}}$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
LGBIP	P_5	F_2	C_6	C_6	I_5	F_{26}	C_6	F_{30}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
line perfect	P_5	F_2	C_6	C_6	I_5	$\lessdot$	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
locally perfect	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
Meyniel	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
murky	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
1-overlap bipartite	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
opposition	P_5	F_2	C_8	C_8	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	F_{24}	$3K_2$	$\overline{C_6}$	C_8	C_8	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
P_4-free	$\lessdot$	F_2	$<$	$<$	I_5	$<$	$<$	F_{19}	$\lessdot$	$<$	$3K_2$	$\lessdot$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	$\lessdot$	$\lessdot$
P_4-lite	P_5	F_2	$<$	$<$	I_5		F_{15}	F_{19}	$2P_4$	$2P_5$	$3K_2$		$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$		$2P_4$
P_4-reducible	F_3	F_2	$<$	$<$	I_5	$<$	$2P_4$	F_{19}	$2P_4$	$<$	$3K_2$		$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$		$2P_4$
P_4-sparse	F_3	F_2	$<$	$<$	I_5		F_{15}	F_{19}	$2P_4$	$<$	$3K_2$		$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$		$2P_4$
P_4-stable Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
parity	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
partner-graph $\triangle$-free	P_5	F_2	C_8	C_8	I_5	F_{31}	F_{15}	F_{19}	$2P_4$	$\overline{F_{31}}$	$3K_2$	$\overline{F_{14}}$	C_8	C_8	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$

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	comparability	$\Delta \leq 6$ Berge	dart-free Berge	degenerate Berge	diamond-free Berge	doc-free Berge	elementary	forest	gem-free Berge	HHD-free	Hoàng	i-triangulated	I_4-free Berge	interval	K_4-free Berge	(K_5, P_5)-free Berge	LGBIP	line perfect	locally perfect	Meyniel
comparability	$=$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
$\Delta \leq 6$ Berge	$\overline{C_6}$	$=$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
dart-free Berge	$\overline{C_6}$	$K_{1,7}$	$=$	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
degenerate Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$=$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
diamond-free Berge	$\overline{C_6}$	$K_{1,7}$	$\lessdot$	$K_{4,4}$	$=$	$\lessdot$	$K_{1,3}$	K_3	$<$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{F_{57}}$	$\overline{P_5}$
doc-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$=$	$K_{1,3}$	K_3	$\lessdot$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{F_{22}}$	$\overline{P_5}$
elementary	$\overline{C_6}$	K_8	$<$	$\overline{4K_2}$	$\overline{F_1}$	$\overline{F_3}$	$=$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$\overline{F_1}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
forest	$<$	$K_{1,7}$	$<$	$<$	$<$	$<$	$K_{1,3}$	$=$	$<$	$<$	F_{35}	$<$	I_4	F_{24}	$<$	P_5	$K_{1,3}$	$<$	$<$	$<$
gem-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	F_8	$K_{1,3}$	K_3	$=$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{F_{22}}$	$\overline{P_5}$
HHD-free	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$=$	F_{15}	$\overline{F_4}$	I_4	C_4	K_4	P_5	$K_{1,3}$	K_5	$<$	$\lessdot$
Hoàng	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	$=$	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
i-triangulated	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	C_6	C_6	$=$	I_4	C_4	K_4	P_5	$K_{1,3}$	K_5	$<$	$\lessdot$
I_4-free Berge	$\overline{C_6}$	K_8	F_6	$\overline{4K_2}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	$=$	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
interval	$\overline{F_{27}}$	$K_{1,7}$	F_6	$<$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$<$	F_{35}	$<$	I_4	$=$	K_4	P_5	$K_{1,3}$	K_5	$<$	$<$
K_4-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	$=$	P_5	$K_{1,3}$	$\overline{P_5}$	$\lessdot$	$\overline{P_5}$
(K_5, P_5)-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	$=$	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
LGBIP	$\overline{C_6}$	K_8	$<$	F_{71}	$\lessdot$	$<$	$\lessdot$	K_3	$<$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$=$	$\overline{P_5}$	$\overline{F_{57}}$	$\overline{P_5}$
line perfect	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\lessdot$	$K_{1,3}$	K_3	$<$	C_6	C_6	$\lessdot$	I_4	C_4	K_4	P_5	$K_{1,3}$	$=$	$<$	$<$
locally perfect	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$=$	$\overline{P_5}$
Meyniel	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	C_6	C_6	$\overline{F_4}$	I_4	C_4	K_4	P_5	$K_{1,3}$	K_5	$\lessdot$	$=$
murky	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{F_{22}}$	$\overline{P_5}$
1-overlap bipartite	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
opposition	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
P_4-free	$<$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	F_8	$K_{1,3}$	K_3	$<$	$<$	$<$	$\overline{F_4}$	I_4	C_4	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
P_4-lite	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$		$\overline{P_5}$
P_4-reducible	$<$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$<$	$\lessdot$	$\overline{F_4}$	I_4	C_4	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
P_4-sparse	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\lessdot$	F_{15}	$\overline{F_4}$	I_4	C_4	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
P_4-stable Berge	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
parity	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	F_8	$K_{1,3}$	K_3	$\lessdot$	C_6	C_6	$\overline{F_4}$	I_4	C_4	K_4	P_5	$K_{1,3}$	K_5	$<$	$\lessdot$
partner-graph $\triangle$-free	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Inclusions between classes of perfect graphs

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	**17**	18
19	20	21	22	23	24

	murky	1-overlap bipartite	opposition	P_4-free	P_4-lite	P_4-reducible	P_4-sparse	P_4-stable Berge	parity	partner-graph △-free	paw-free Berge	perfectly contractile	perfectly orderable	permutation	planar Berge	preperfect	quasi-parity	Raspail	skeletal	slender
comparability	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{23}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$<$	$\lessdot$	C_6	K_5		$<$	$\lessdot$	$\overline{F_{16}}$	$\overline{F_{19}}$
$\Delta \leq 6$ Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
dart-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
degenerate Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
diamond-free Berge	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	F_{26}
doc-free Berge	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
elementary	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\lessdot$	$\overline{C_6}$	$\overline{F_{19}}$
forest	P_6	F_{47}	F_{35}	P_4	P_6	P_5	P_5	$<$	$<$	F_9	$<$	$<$	$<$	F_{24}	$\lessdot$	$<$	$<$	$<$	$<$	$<$
gem-free Berge	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
HHD-free	P_6	$\overline{F_{27}}$	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	F_9	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$	F_{56}	$\overline{F_{15}}$	$\overline{F_{19}}$
Hoàng	P_6	F_{31}	$\overline{F_{24}}$	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
i-triangulated	P_6	$\overline{F_{27}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	C_6	$\overline{F_2}$	$<$	F_{73}	C_6	K_5	$<$	$<$	$\lessdot$	$\overline{F_{15}}$	$\lessdot$
I_4-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
interval	P_6	$\overline{F_{27}}$	F_{35}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	F_9	$\overline{F_2}$	$<$	$<$	$\overline{F_{27}}$	K_5	$<$	$<$	$<$	$\overline{F_{27}}$	$<$
K_4-free Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	$K_{3,3}$	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	F_{29}
(K_5, P_5)-free Berge	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	$K_{3,3}$		F_{68}	F_{29}	$\overline{C_6}$	F_{29}
LGBIP	P_6	F_{46}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$<$	$\overline{C_6}$	F_{26}
line perfect	P_6	F_{34}	C_6	P_4	P_6	P_5	P_5	F_{58}	$\lessdot$	C_6	$\overline{F_2}$	$<$	F_{73}	C_6	$K_{3,3}$	$<$	$<$	$<$	$<$	$<$
locally perfect	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
Meyniel	P_6	$\overline{F_{27}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	C_6	$\overline{F_2}$	$\lessdot$	F_{67}	C_6	K_5		$<$	F_{56}	$\overline{F_{15}}$	$\overline{F_{19}}$
murky	$=$	F_{27}	C_6	P_4	C_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
1-overlap bipartite	P_6	$=$	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5			$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
opposition	P_6	F_{31}	$=$	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$\overline{F_{74}}$	F_{68}	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
P_4-free	$<$	$<$	$<$	$=$	$<$	$\lessdot$	$<$	$<$	$\lessdot$	$<$	$\overline{F_2}$	$<$	$<$	$<$	K_5	$<$	$<$	$<$	$<$	$\overline{F_{19}}$
P_4-lite	$\lessdot$			P_4	$=$	P_5	P_5	$\overline{2P_4}$	$\overline{P_5}$	$\lessdot$	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$		$\overline{F_{15}}$	$\overline{F_{19}}$
P_4-reducible	$<$	$<$	$\lessdot$	P_4	$<$	$=$	$\lessdot$	$\overline{2P_4}$	$\overline{F_3}$	$<$	$\overline{F_2}$	$<$	$<$	$\lessdot$	K_5	$<$	$<$	$<$	$\overline{2P_4}$	$\overline{F_{19}}$
P_4-sparse	$<$	$\lessdot$		P_4	$\lessdot$	F_{15}	$=$	$\overline{2P_4}$	$\overline{F_3}$	$<$	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$		$\overline{F_{15}}$	$\overline{F_{19}}$
P_4-stable Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$=$	$\overline{P_5}$	C_6	$\overline{F_2}$	F_{44}	$\overline{F_{42}}$	C_6	K_5		F_{59}	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
parity	P_6	F_{34}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{36}}$	$=$	C_6	$\overline{F_2}$	$<$	F_{73}	C_6	K_5	$\lessdot$	$<$	F_{56}	$\lessdot$	$\overline{F_{19}}$
partner-graph △-free	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	$=$	$\overline{F_2}$	$\overline{C_8}$	$\overline{C_8}$	F_{15}	K_5		$\lessdot$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$

Inclusions between classes of perfect graphs

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	slightly triangulated	slim	snap	split	strict opposition	strict quasi-parity	strongly perfect	3-overlap bipartite	3-overlap △-free	threshold	tree	triangulated	trivially perfect	2-overlap bipartite	2-overlap △-free	2-split Berge	$2K_2$-free Berge	unimodular	weakly triangulated	wing triangulated
comparability	C_6	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
$\Delta \leq 6$ Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
dart-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
degenerate Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
diamond-free Berge	C_6	$\overline{C_6}$	F_{60}	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	F_{54}	C_6	$\overline{C_6}$
doc-free Berge	C_6	$\overline{C_6}$	$\overline{2C_4}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	F_{40}	C_6	$\overline{C_6}$
elementary	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
forest	$<$	$<$	$<$	$\overline{C_4}$	F_{35}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	$\lessdot$	P_4	F_{24}	F_{24}	$<$	$\overline{C_4}$	$<$	$<$	F_{10}
gem-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
HHD-free	$\overline{2P_4}$	$<$	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$<$	F_{10}
Hoàng	C_8	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
i-triangulated	C_6	$<$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
I_4-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_9}$	$\overline{F_9}$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
interval	$<$	$<$	$<$	$\overline{C_4}$	F_{35}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	$\lessdot$	P_4	$\overline{F_{14}}$	$\overline{F_{14}}$	$3K_3$	$\overline{C_4}$	$\lessdot$	$<$	F_{10}
K_4-free Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
(K_5, P_5)-free Berge	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
LGBIP	C_6	$\overline{C_6}$	F_{60}	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$\overline{F_{12}}$	$\overline{F_{12}}$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\lessdot$	C_6	$\overline{C_6}$
line perfect	C_6	$<$	$\lessdot$	$\overline{C_4}$	C_6	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$		C_6	F_{10}
locally perfect	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
Meyniel	C_6	$\lessdot$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$\lessdot$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
murky	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
1-overlap bipartite	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
opposition	C_8	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
P_4-free	$\lessdot$	$<$	$\overline{3K_2}$	$\overline{C_4}$	$\lessdot$	$<$	$<$	$<$	$<$	$\overline{C_4}$	I_2	C_4	C_4	$<$	$<$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$<$	$\lessdot$
P_4-lite	$\overline{2P_4}$	$\overline{2P_5}$	$\overline{3K_2}$	$\overline{C_4}$		$<$	$<$	$\lessdot$	$<$	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$<$	
P_4-reducible	$\overline{2P_4}$	$<$	$\overline{3K_2}$	$\overline{C_4}$		$<$	$<$	$<$	$<$	$\overline{C_4}$	I_2	C_4	P_4	$\lessdot$	$<$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$<$	
P_4-sparse	$\overline{2P_4}$	$<$	$\overline{3K_2}$	$\overline{C_4}$		$<$	$<$	$<$	$<$	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$<$	
P_4-stable Berge	C_6	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	C_6	F_{59}	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
parity	C_6	$<$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
partner-graph △-free	C_8	F_{31}	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$\overline{C_8}$	$\overline{C_8}$	F_{18}	$\lessdot$	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_8	F_{10}

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

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	alternately colorable	alternately orientable	AT-free Berge	BIP*	bipartite	brittle	bull-free Berge	C_4-free Berge	chair-free Berge	claw-free Berge	clique-separable	co-alternately colorable	co-alternately orientable	co-AT-free Berge	co-BIP*	co-bipartite	co-chair-free Berge	co-claw-free Berge	co-clique-separable	co-cograph contraction
paw-free Berge	$K_{2,3}$	$<$	C_6	$<$	K_3	C_6	$\lessdot$	C_4	$\overline{F_7}$	$K_{1,3}$	$<$	$\overline{K_{2,3}}$	C_6	$<$	C_6	I_3	$\lessdot$	$\overline{K_{1,3}}$	F_4	P_6
perfectly contractile	$K_{2,3}$	F_{41}	C_6	F_{55}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
perfectly orderable	$K_{2,3}$	F_{41}	C_6	$\lessdot$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
permutation	$K_{2,3}$	$<$	$<$	$<$	K_3	F_{38}	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$<$	$<$	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
planar Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
preperfect	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
quasi-parity	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
Raspail	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
skeletal	$K_{2,3}$	F_{41}	C_6		K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
slender	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
slightly triangulated	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	F_{41}	$\overline{C_6}$	$\overline{F_{44}}$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
slim	$K_{2,3}$	F_{41}	C_6		K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
snap	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
split	$<$	$<$	F_{15}	$<$	K_3	$<$	F_5	$<$	$\overline{F_7}$	$K_{1,3}$	$<$	$<$	$<$	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	$<$	$\lessdot$
strict opposition	$K_{2,3}$	F_{41}	F_{15}	$\lessdot$	K_3	C_8	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_8	F_{15}	C_8	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
strict quasi-parity	$K_{2,3}$	F_{41}	C_6	F_{44}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
strongly perfect	$K_{2,3}$	F_{41}	C_6	F_{55}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
3-overlap bipartite	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
3-overlap △-free	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
threshold	$<$	$<$	$<$	$<$	K_3	$<$	$<$	$<$	$<$	$K_{1,3}$	$<$	$<$	$<$	$<$	$<$	I_3	$<$	$\overline{K_{1,3}}$	$<$	$<$
tree	$<$	$<$	F_{24}	$<$	$<$	$<$	$<$	$<$	$\overline{F_7}$	$K_{1,3}$	$<$	$<$	F_{61}	$<$	$<$	$K_{1,3}$	$<$	$<$	P_6	P_6
triangulated	$\lessdot$	$<$	F_{15}	$<$	K_3	$<$	F_5	$\lessdot$	$\overline{F_7}$	$K_{1,3}$	$<$	$\overline{K_{2,3}}$	$\overline{F_{56}}$	F_{15}	$<$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
trivially perfect	$<$	$<$	$<$	$<$	K_3	$<$	$<$	$<$	$<$	$K_{1,3}$	$<$	$\overline{K_{2,3}}$	$<$	$<$	$<$	I_3	$<$	$\overline{K_{1,3}}$	F_4	$<$
2-overlap bipartite	$K_{2,3}$	$\overline{C_8}$	$\overline{F_{17}}$	$\overline{C_8}$	K_3	C_8	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_8	F_{17}	C_8	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
2-overlap △-free	$K_{2,3}$	$\overline{C_8}$	$\overline{F_{17}}$	$\overline{C_8}$	K_3	C_8	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_8	F_{17}	C_8	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
2-split Berge	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
$2K_2$-free Berge	$K_{2,3}$	$\overline{C_6}$	F_{15}	$\overline{C_6}$	K_3	$\overline{C_6}$	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{F_{43}}$	F_{55}	$\overline{C_6}$	$\overline{F_{55}}$	I_3	F_7	$\overline{K_{1,3}}$	F_{29}	$\overline{C_6}$
unimodular	$K_{2,3}$	$\overline{C_6}$	C_6	$\overline{C_6}$	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	$\overline{C_6}$	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
weakly triangulated	$K_{2,3}$	F_{41}	F_{15}	$\lessdot$	K_3	F_{42}	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	$\overline{F_{41}}$	F_{15}	$\lessdot$	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6
wing triangulated	$K_{2,3}$	$\overline{F_{45}}$	C_6	F_{55}	K_3	C_6	F_5	C_4	$\overline{F_7}$	$K_{1,3}$	$\overline{F_4}$	$\overline{K_{2,3}}$	C_6	F_{15}	C_6	I_3	F_7	$\overline{K_{1,3}}$	F_4	P_6

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	co-comparability	co-$\Delta \leq 6$ Berge	co-dart-free Berge	co-degenerate Berge	co-diamond-free Berge	co-doc-free Berge	co-elementary	co-forest	co-gem-free Berge	co-HHD-free	co-Hoàng	co-i-triangulated	co-interval	co-(K_5, P_5)-free Berge	co-LGBIP	co-line perfect	co-locally perfect	co-Meyniel	co-opposition	co-P_4-stable Berge
paw-free Berge	C_6	I_8	$\lessdot$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\lessdot$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{24}	C_6
perfectly contractile	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
perfectly orderable	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
permutation	$\lessdot$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{14}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	F_{23}
planar Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
preperfect	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
quasi-parity	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
Raspail	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
skeletal	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{14}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
slender	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
slightly triangulated	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	F_{27}
slim	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
snap	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
split	F_{15}	I_8	$\overline{F_6}$	$<$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$<$	$\overline{F_{15}}$	$<$	F_{15}	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$\lessdot$	$<$
strict opposition	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	F_{23}
strict quasi-parity	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
strongly perfect	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6
3-overlap bipartite	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
3-overlap △-free	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
threshold	$<$	I_8	$\overline{F_6}$	$<$	F_1	$<$	$\overline{K_{1,3}}$	I_3	$<$	$<$	$<$	$<$	$<$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$<$	$<$
tree	F_{24}	$K_{1,8}$	$<$	F_{48}	$\overline{F_7}$	P_6	$<$	$K_{1,3}$	P_6	P_5	$<$	P_5	P_5	$K_{1,5}$	$\overline{F_7}$	P_5	P_7	P_5	F_{24}	P_9
triangulated	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	$2P_4$
trivially perfect	$<$	I_8	$\overline{F_6}$	$4K_2$	F_1	$\overline{F_8}$	$\overline{K_{1,3}}$	I_3	$<$	$<$	$<$	F_4	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	$<$	$<$	$<$	$<$
2-overlap bipartite	$\overline{F_{17}}$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5		P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5		$2P_4$
2-overlap △-free	$\overline{F_{17}}$	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	F_{52}	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	F_{52}	$2P_4$
2-split Berge	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
$2K_2$-free Berge	F_{15}	I_8	$\overline{F_6}$	$\overline{F_{72}}$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	$\overline{C_6}$	$\overline{C_6}$	F_{29}	$\overline{C_6}$	I_5	$\overline{K_{1,3}}$	I_5	F_{65}	$\overline{F_{26}}$	$\overline{C_6}$	F_{27}
unimodular	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{C_6}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{C_6}$	C_6
weakly triangulated	F_{15}	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	F_{27}
wing triangulated	C_6	I_8	$\overline{F_6}$	$4K_2$	F_1	F_3	$\overline{K_{1,3}}$	I_3	F_3	P_5	$\overline{F_{15}}$	P_5	$\overline{C_4}$	I_5	$\overline{K_{1,3}}$	I_5	P_7	P_5	$\overline{F_{14}}$	C_6

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	co-parity	co-paw-free Berge	co-perfectly contractile	co-perfectly orderable	co-planar Berge	co-Raspail	co-skeletal	co-slender	co-slightly triangulated	co-slim	co-snap	co-strict opposition	co-strict quasi-parity	co-strongly perfect	co-tree	co-triangulated	co-trivially perfect	co-unimodular	co-wing triangulated	cograph contraction
paw-free Berge	P_5	F_2	C_6	C_6	I_5		C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
perfectly contractile	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
perfectly orderable	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
permutation	P_5	F_2	$<$	$<$	I_5	$<$	F_{16}	F_{19}	$2P_4$	F_{22}	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
planar Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
preperfect	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
quasi-parity	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
Raspail	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
skeletal	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
slender	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
slightly triangulated	P_5	F_2	$\overline{F_{44}}$	F_{42}	I_5	F_{31}	F_{15}	F_{19}	$\overline{C_6}$	$\overline{F_{31}}$	$3K_2$	$\overline{C_6}$	$\overline{F_{64}}$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{C_6}$
slim	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
snap	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
split	F_3	F_2	$<$	$<$	I_5	$<$	F_{15}	$<$	$<$	$<$	$<$		$<$	$<$	K_2	$\lessdot$	P_4	F_{15}	F_{10}	$\lessdot$
strict opposition	P_5	F_2	C_8	C_8	I_5	F_{31}	F_{15}	F_{19}	$2P_4$	F_{24}	$3K_2$	$\overline{F_{14}}$	C_8	C_8	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
strict quasi-parity	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
strongly perfect	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
3-overlap bipartite	P_5	F_2	C_6	C_6	I_5	$\overline{F_{28}}$	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
3-overlap △-free	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
threshold	$<$	F_2	$<$	$<$	I_5	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	K_2	$<$	$\lessdot$	$<$	$<$	$<$
tree	P_5	P_5	$<$	$<$	$K_{1,5}$	$<$	P_8	F_{20}	P_9	F_{24}	F_{24}	F_{24}	$<$	$<$	K_2	P_5	P_4	F_{13}	F_{10}	P_7
triangulated	P_5	F_2	$<$	$<$	I_5	$\overline{F_{56}}$	F_{15}	F_{19}	$2P_4$	F_{24}	$3K_2$	$\overline{F_{14}}$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	P_7
trivially perfect	$<$	F_2	$<$	$<$	I_5	$<$	$<$	F_{19}	$<$	$<$	$3K_2$	$<$	$<$	$<$	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	$<$	$<$
2-overlap bipartite	P_5	F_2	C_8	C_8	I_5		F_{25}	F_{19}	$2P_4$	C_8	$3K_2$	C_8	C_8	C_8	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
2-overlap △-free	P_5	F_2	C_8	C_8	I_5	F_{50}	F_{25}	F_{19}	$2P_4$	C_8	$3K_2$	C_8	C_8	C_8	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
2-split Berge	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
$2K_2$-free Berge	F_3	F_2	$\overline{F_{62}}$	$\overline{F_{55}}$	I_5	$\overline{F_{66}}$	F_{15}	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{55}}$	F_{69}	$\overline{C_6}$	$\overline{F_{68}}$	$\overline{F_{62}}$	K_2	$\overline{C_6}$	P_4	F_{15}	F_{10}	$\overline{C_6}$
unimodular	P_5	F_2	C_6	C_6	I_5	F_{31}	C_6	F_{19}	$\overline{C_6}$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6
weakly triangulated	P_5	F_2	$\lessdot$	F_{42}	I_5	F_{31}	F_{15}	F_{19}	$2P_4$	$\overline{F_{31}}$	$3K_2$	$\overline{F_{14}}$	$<$	F_{42}	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	F_{10}	$\overline{P_6}$
wing triangulated	P_5	F_2	C_6	C_6	I_5	F_{29}	C_6	F_{19}	$2P_4$	C_6	$3K_2$	C_6	C_6	C_6	K_2	$\overline{C_4}$	$\overline{C_4}$	$3K_2$	C_6	C_6

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	comparability	$\Delta \leq 6$ Berge	dart-free Berge	degenerate Berge	diamond-free Berge	doc-free Berge	elementary	forest	gem-free Berge	HHD-free	Hoàng	i-triangulated	I_4-free Berge	interval	K_4-free Berge	(K_5, P_5)-free Berge	LGBIP	line perfect	locally perfect	Meyniel
paw-free Berge	$\lessdot$	$K_{1,7}$	$\lessdot$	$K_{4,4}$	$\overline{F_1}$	F_8	$K_{1,3}$	K_3	$<$	C_6	C_6	$\lessdot$	I_4	C_4	K_4	P_5	$K_{1,3}$	K_5	$<$	$<$
perfectly contractile	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
perfectly orderable	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
permutation	$\lessdot$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{14}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
planar Berge	$\overline{C_6}$	$K_{1,7}$	F_6		$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$		$\overline{P_5}$
preperfect	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
quasi-parity	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
Raspail	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
skeletal	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
slender	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
slightly triangulated	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
slim	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
snap	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
split	F_{15}	$K_{1,7}$	F_6	$<$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$<$	F_{15}	$<$	I_4	F_{15}	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
strict opposition	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
strict quasi-parity	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
strongly perfect	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
3-overlap bipartite	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
3-overlap △-free	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
threshold	$<$	$K_{1,7}$	F_6	$<$	$\overline{F_1}$	$<$	$K_{1,3}$	K_3	$<$	$<$	$<$	$<$	I_4	$<$	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
tree	$<$	$K_{1,7}$	$<$	$<$	$<$	$<$	$K_{1,3}$	$\lessdot$	$<$	$<$	F_{35}	$<$	$K_{1,4}$	F_{24}	$<$	P_5	$K_{1,3}$	$<$	$<$	$<$
triangulated	F_{15}	$K_{1,7}$	F_6	$\lessdot$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\lessdot$	F_{15}	$\lessdot$	I_4	F_{15}	K_4	P_5	$K_{1,3}$	K_5	$<$	$<$
trivially perfect	$<$	$K_{1,7}$	F_6	$<$	$\overline{F_1}$	$\lessdot$	$K_{1,3}$	K_3	$<$	$<$	$<$	$<$	I_4	$\lessdot$	K_4	K_5	$K_{1,3}$	K_5	$<$	$<$
2-overlap bipartite	F_{17}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$		$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
2-overlap △-free	F_{17}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{52}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
2-split Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
$2K_2$-free Berge	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	K_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
unimodular	$\overline{C_6}$	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	F_{22}	$\overline{P_5}$
weakly triangulated	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	F_{15}	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$
wing triangulated	F_{15}	$K_{1,7}$	F_6	$K_{4,4}$	$\overline{F_1}$	$\overline{F_3}$	$K_{1,3}$	K_3	$\overline{F_3}$	$\overline{P_5}$	C_6	$\overline{P_5}$	I_4	C_4	K_4	P_5	$K_{1,3}$	$\overline{P_5}$	$\overline{P_7}$	$\overline{P_5}$

This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Inclusions between classes of perfect graphs

1	2	3	4	5	6
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19	20	21	22	**23**	24

	murky	1-overlap bipartite	opposition	P_4-free	P_4-lite	P_4-reducible	P_4-sparse	P_4-stable Berge	parity	partner-graph △-free	paw-free Berge	perfectly contractile	perfectly orderable	permutation	planar Berge	preperfect	quasi-parity	Raspail	skeletal	slender
paw-free Berge	P_6	F_{34}	C_6	P_4	P_6	P_5	P_5	$\lessdot$	$\lessdot$	C_6	$=$	$<$	$<$	C_6	K_5	$<$	$<$	$<$	$<$	$<$
perfectly contractile	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$=$	$\overline{F_{42}}$	C_6	K_5		$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
perfectly orderable	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\lessdot$	$=$	C_6	K_5		$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
permutation	P_6	F_{34}	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{23}}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$<$	$<$	$=$	K_5		$<$	$<$	$\overline{F_{16}}$	$\overline{F_{19}}$
planar Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	$=$	F_{70}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	F_{29}
preperfect	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	$=$	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
quasi-parity	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5		$=$	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
Raspail	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$=$	$\overline{C_6}$	$\overline{F_{19}}$
skeletal	P_6	F_{27}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{34}}$	$\overline{P_5}$	C_6	$\overline{F_2}$		F_{63}	C_6	K_5		$<$	F_{28}	$=$	$\overline{F_{19}}$
slender	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$=$
slightly triangulated	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5			$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
slim	P_6	$\overline{F_{31}}$	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$		F_{54}	C_6	K_5			$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
snap	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5		F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
split	$\lessdot$	F_{33}	$\lessdot$	P_4	F_9	F_7	F_7	$<$	$\overline{F_3}$	F_9	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$	$<$	$\overline{F_{15}}$	$<$
strict opposition	P_6	F_{31}	$\lessdot$	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	F_9	$\overline{F_2}$			F_{15}	K_5		$<$	F_{28}	$\overline{F_{15}}$	$\overline{F_{19}}$
strict quasi-parity	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$	F_{44}	$\overline{F_{42}}$	C_6	K_5		$\lessdot$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
strongly perfect	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	C_6	$\overline{F_2}$		F_{55}	C_6	K_5			$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
3-overlap bipartite	P_6	F_{46}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	F_{28}	$\overline{C_6}$	$\overline{F_{19}}$
3-overlap △-free	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
threshold	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$\overline{F_2}$	$<$	$<$	$<$	K_5	$<$	$<$	$<$	$<$	$<$
tree	P_6	F_{47}	F_{35}	P_4	P_6	P_5	P_5	$<$	$<$	F_9	$<$	$<$	$<$	F_{24}	$<$	$<$	$<$	$<$	$<$	$<$
triangulated	P_6	$\overline{F_{27}}$	F_{35}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{F_3}$	F_9	$\overline{F_2}$	$<$	$<$	F_{15}	K_5	$<$	$<$	$<$	$\overline{F_{15}}$	$<$
trivially perfect	$<$	$<$	$<$	$\lessdot$	$<$	$<$	$<$	$<$	$<$	$<$	$\overline{F_2}$	$<$	$<$	$<$	K_5	$<$	$<$	$<$	$<$	$<$
2-overlap bipartite	P_6	F_{37}		P_4	P_6	P_5	P_5	$\overline{C_8}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$\overline{C_8}$	$\overline{C_8}$	F_{17}	K_5		$<$		$\overline{F_{25}}$	$\overline{F_{19}}$
2-overlap △-free	P_6	F_{37}	$\overline{F_{52}}$	P_4	P_6	P_5	P_5	$\overline{C_8}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$\overline{C_8}$	$\overline{C_8}$	F_{17}	K_5		$\lessdot$	$\overline{F_{50}}$	$\overline{F_{25}}$	$\overline{F_{19}}$
2-split Berge	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
$2K_2$-free Berge	$\overline{P_6}$	F_{27}	F_{14}	P_4	$\overline{C_6}$	$\overline{P_5}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{P_5}$	$\overline{C_6}$	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	K_5	$\overline{F_{74}}$	$\overline{F_{68}}$	$\overline{F_{29}}$	$\overline{C_6}$	$\overline{F_{19}}$
unimodular	P_6	F_{31}	C_6	P_4	P_6	P_5	P_5	$\overline{C_6}$	$\overline{P_5}$	C_6	$\overline{F_2}$	$\overline{C_6}$	$\overline{C_6}$	C_6	K_5	F_{60}	F_{46}	$\overline{F_{31}}$	$\overline{C_6}$	$\overline{F_{19}}$
weakly triangulated	P_6	F_{31}	F_{14}	P_4	P_6	P_5	P_5	$\overline{F_{27}}$	$\overline{P_5}$	F_9	$\overline{F_2}$	$\lessdot$	$\overline{F_{42}}$	F_{15}	K_5		$<$	$\overline{F_{31}}$	$\overline{F_{15}}$	$\overline{F_{19}}$
wing triangulated	P_6	F_{37}	C_6	P_4	P_6	P_5	P_5	$2P_4$	$\overline{P_5}$	C_6	$\overline{F_2}$		F_{55}	C_6	K_5		$<$	F_{53}	$\overline{F_{15}}$	$\overline{F_{19}}$

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Inclusions between classes of perfect graphs	slightly triangulated	slim	snap	split	strict opposition	strict quasi-parity	strongly perfect	3-overlap bipartite	3-overlap △-free	threshold	tree	triangulated	trivially perfect	2-overlap bipartite	2-overlap △-free	2-split Berge	$2K_2$-free Berge	unimodular	weakly triangulated	wing triangulated
paw-free Berge	C_6	$<$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
perfectly contractile	C_6	$\overline{F_{31}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\lessdot$	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
perfectly orderable	C_6	$\overline{F_{31}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$<$	$\lessdot$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
permutation	$\overline{2P_4}$	$\overline{F_{22}}$	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{14}	F_{14}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\lessdot$	F_{10}
planar Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
preperfect	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
quasi-parity	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
Raspail	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
skeletal	C_6	$\overline{F_{24}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\lessdot$	F_{63}	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
slender	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
slightly triangulated	$=$	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
slim	C_6	$=$	$\overline{3K_2}$	$\overline{C_4}$	C_6		F_{54}	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
snap	C_6	$\overline{C_6}$	$=$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{F_{15}}$	C_6	$\overline{C_6}$
split	$<$	$<$	$<$	$=$		$<$	$<$	F_9	F_9	P_4	I_2	$\lessdot$	P_4	F_{15}	F_{15}	$\lessdot$	$<$	$\overline{F_{15}}$	$<$	F_{10}
strict opposition	C_8	$\overline{F_{31}}$	$\overline{3K_2}$	$\overline{C_4}$	$=$	$\lessdot$		F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_8	F_{10}
strict quasi-parity	C_6	$\overline{F_{31}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$=$	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
strongly perfect	C_6	$\overline{F_{31}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6		$=$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	F_{10}
3-overlap bipartite	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	$=$	$\lessdot$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
3-overlap △-free	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_{18}	$=$	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
threshold	$<$	$<$	$<$	$\lessdot$	$<$	$<$	$<$	$<$	$<$	$=$	I_2	$<$	$\lessdot$	$<$	$<$	$<$	$<$	$<$	$<$	$<$
tree	$<$	$<$	$<$	P_5	F_{35}	$<$	$<$	F_9	F_9	P_4	$=$	$<$	P_4	F_{24}	F_{24}	$<$	P_5	$<$	$<$	F_{10}
triangulated	$\lessdot$	$<$	$\lessdot$	$\overline{C_4}$	F_{35}	$<$	$<$	F_9	F_9	$\overline{C_4}$	I_2	$=$	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{F_{15}}$	$<$	F_{10}
trivially perfect	$<$	$<$	$<$	$\overline{C_4}$	$<$	$<$	$<$	$<$	$<$	$\overline{C_4}$	I_2	$<$	$=$	$<$	$<$	$3K_3$	$\overline{C_4}$	$<$	$<$	$<$
2-overlap bipartite	C_8	$\overline{C_8}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_8}$	$\overline{C_8}$	$\overline{C_8}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	$=$	$\lessdot$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_8	F_{10}
2-overlap △-free	C_8	$\overline{C_8}$	$\overline{3K_2}$	$\overline{C_4}$	$\overline{C_8}$	$\overline{C_8}$	$\overline{C_8}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{18}	$=$	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_8	F_{10}
2-split Berge	C_6	$\overline{C_6}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$=$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$\overline{C_6}$
$2K_2$-free Berge	$\overline{C_8}$	$\overline{C_6}$	$\overline{3K_2}$	C_4	$\overline{C_6}$	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	P_4	I_2	C_4	P_4	$\overline{C_6}$	$\overline{C_6}$	$\overline{3K_3}$	$=$	$\overline{3K_2}$	$\overline{C_6}$	$\overline{C_6}$
unimodular	C_6	$\overline{C_6}$	$\overline{2C_4}$	$\overline{C_4}$	C_6	$\overline{C_6}$	$\overline{C_6}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$=$	C_6	$\overline{C_6}$
weakly triangulated	$\overline{2P_4}$	$\overline{F_{31}}$	$\overline{3K_2}$	$\overline{C_4}$	F_{14}	$<$	$\overline{F_{42}}$	F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	F_{15}	F_{15}	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	$=$	F_{10}
wing triangulated	C_6	$\overline{F_{32}}$	$\overline{3K_2}$	$\overline{C_4}$	C_6	$\lessdot$		F_9	F_9	$\overline{C_4}$	I_2	C_4	P_4	C_6	C_6	$3K_3$	$\overline{C_4}$	$\overline{3K_2}$	C_6	$=$

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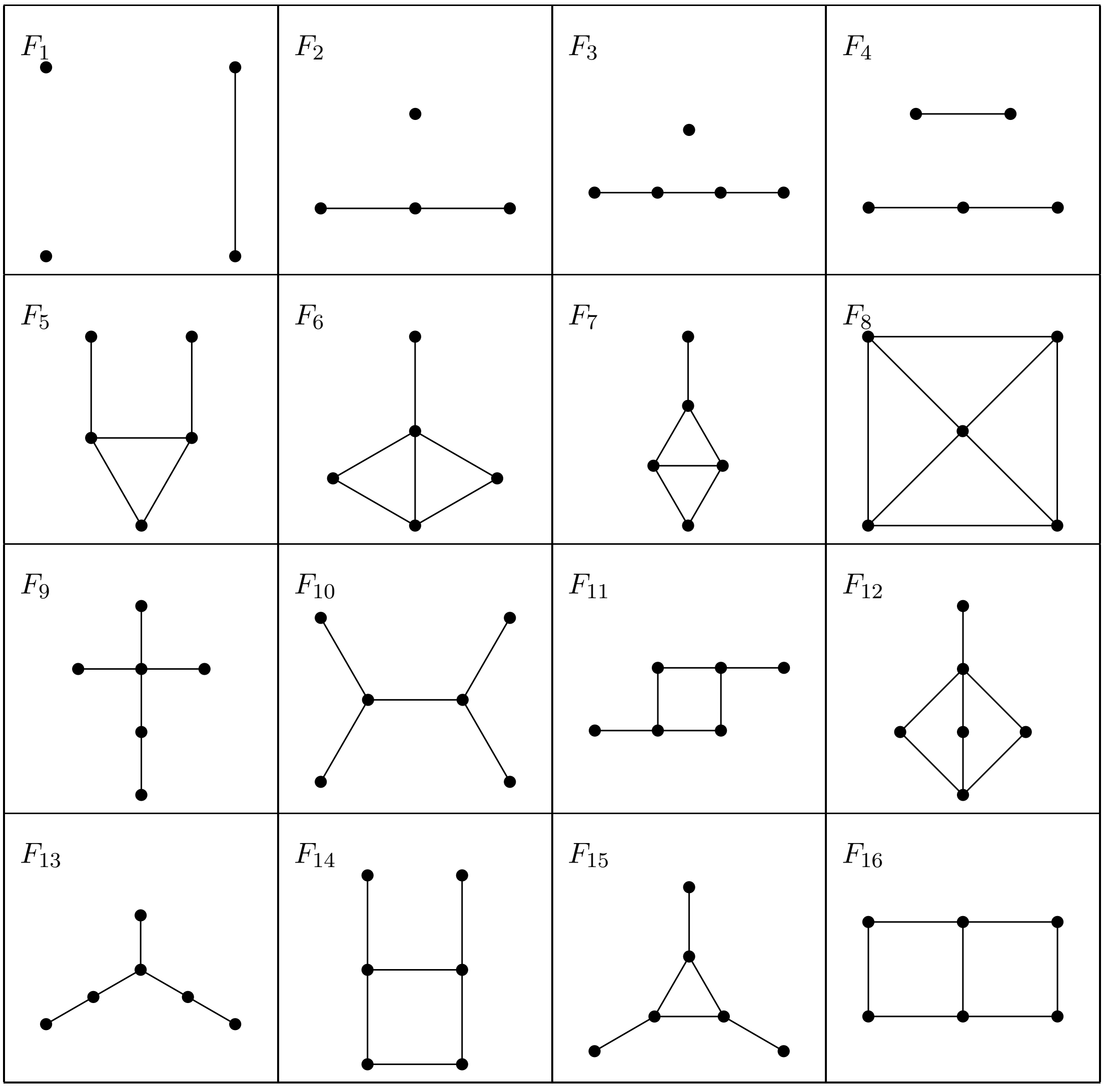
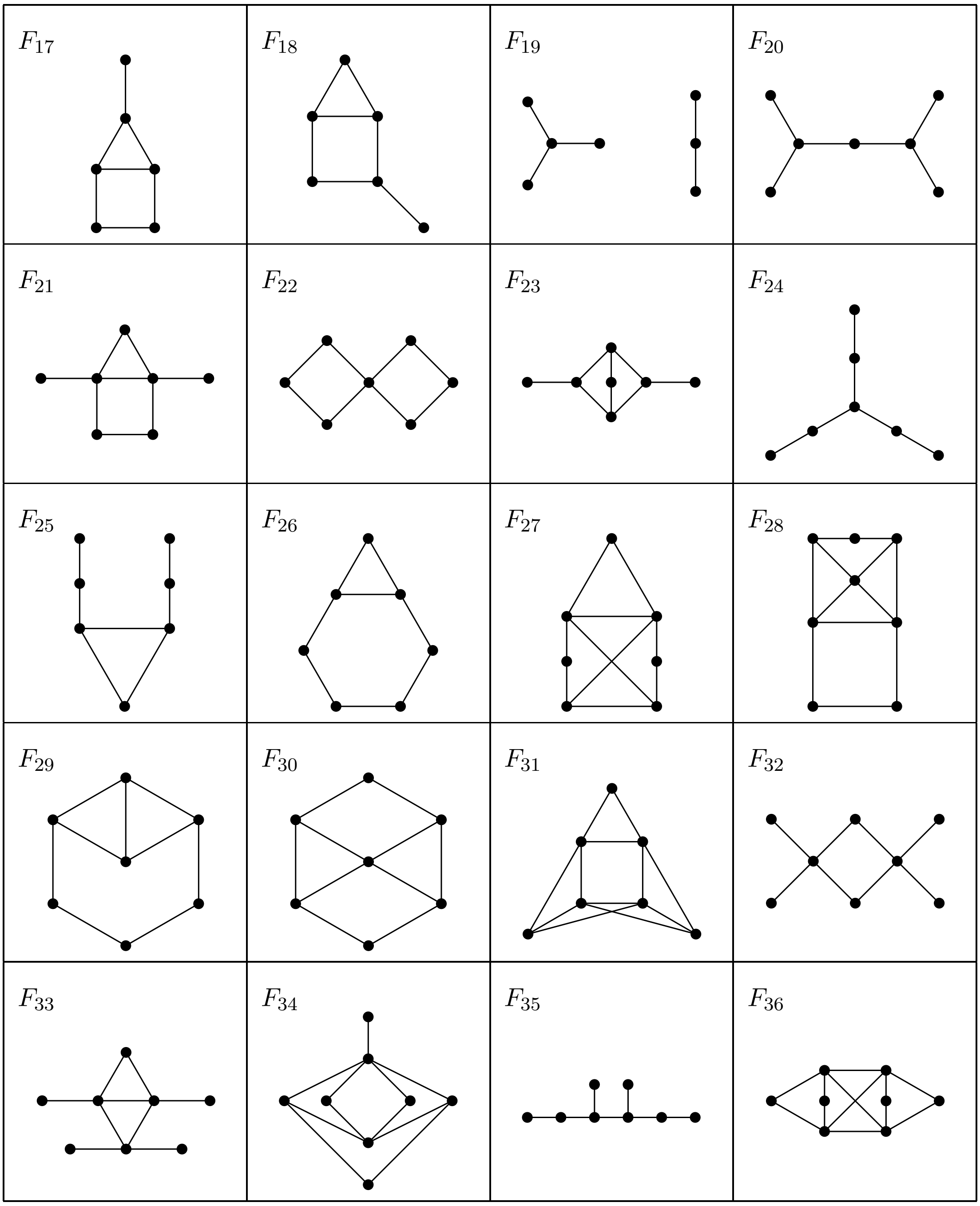
7 Counterexamples

The counterexamples F_i appearing in the previous table to prove that some class is not contained in some other class are shown in this section. Fortunately, while we give 12888 such counterexamples, it turns out that only 74 *different* counterexamples are needed.



This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571



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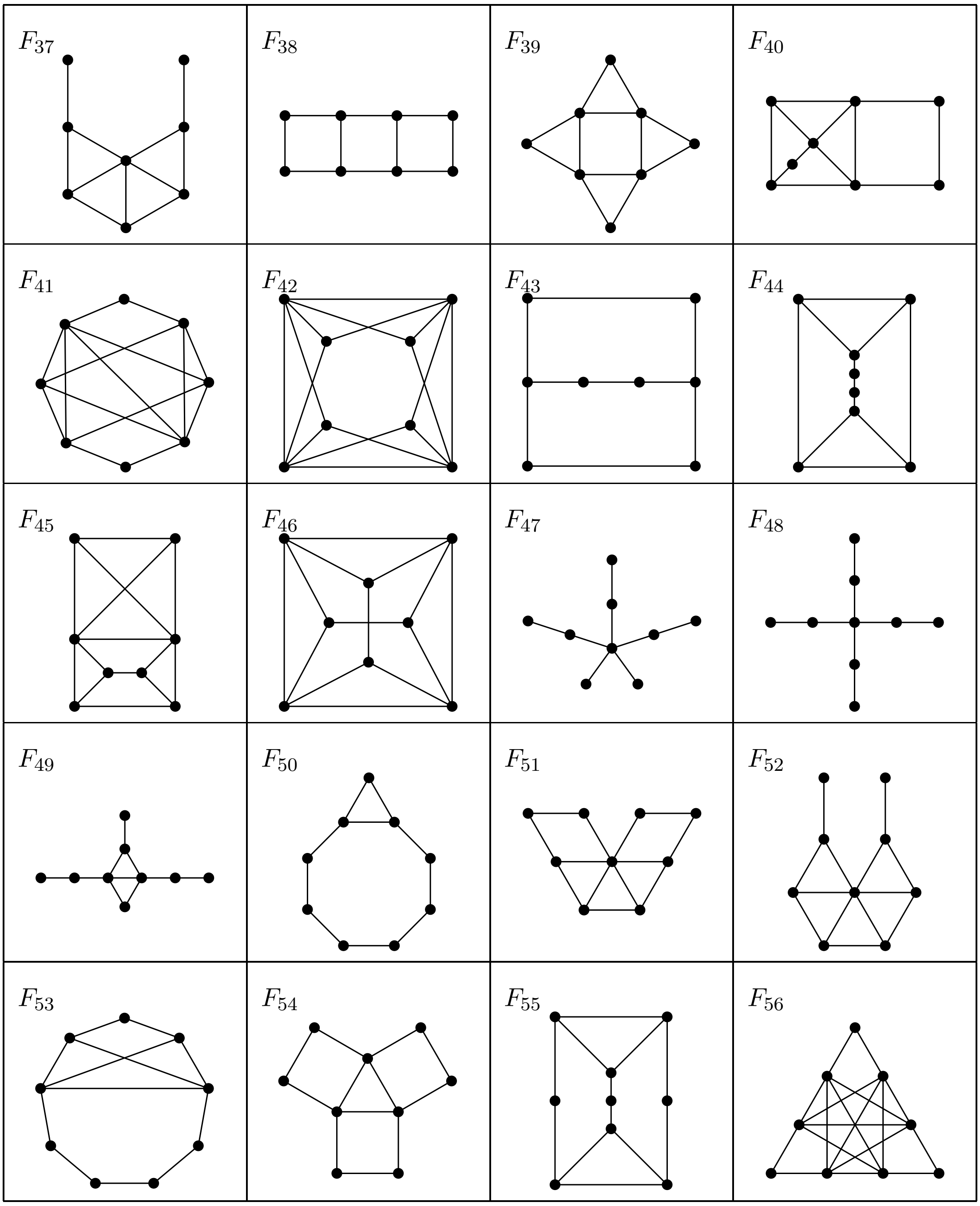

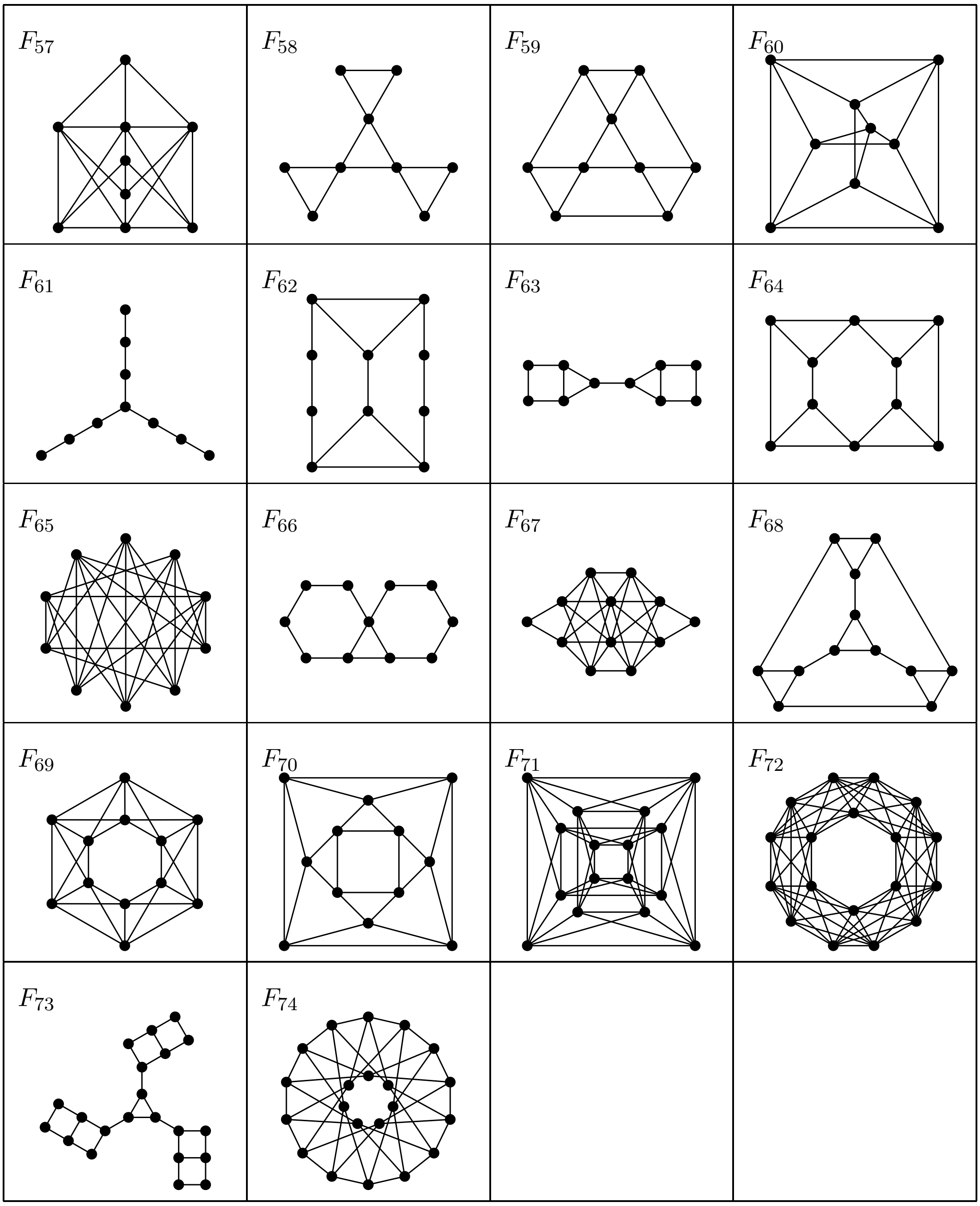
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This paper appeared in: Discrete Mathematics 306 (2006), 2529-2571

Acknowledgement

I am grateful to Mariano Zelke and Valentin Ziegler for some help in preparing the final version of this paper. I thank Adrian Bondy, Chính T. Hoàng, Van Bang Le, Frédéric Maffray, Annegret Wagler and two anonymous referees for useful comments and I thank Vašek Chvátal for suggestions improving the presentation of this paper.

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