Prof. Dr. Jens Vygen
Dr. Stephan Held

Exercises 12

1. Consider the first step of the algorithm for the Sink Clustering Problem. Assume that a minimum spanning tree in (D, c) can be computed in $O(n \log n)$ time (which is possible for finite sets of points in the rectilinear plane). Prove that one can then compute $F_{t''}$ in $O(n \log n)$ time. [5 points]

2. Formulate the Simple Global Routing Problem as an integer linear program (ILP) with a polynomial number of variables and constraints. [3 points]

3. Given an instance of the Simple Global Routing Problem and additionally a list $\mathcal{P}$ of timing critical paths with delay bounds $D : \mathcal{P} \to \mathbb{R}_+$. Each path $P \in \mathcal{P}$ consists of a sequence $C_1, N_1, C_2, N_2, \ldots, C_{n_P-1}, N_{n_P-1}, C_{n_P}$ of circuits $C_i \in \mathcal{C}$ and nets $N_i \in \mathcal{N}$ in the netlist. Let $\mathcal{Y}_N$ be the set of Steiner trees for $N \in \mathcal{N}$. A delay function $d_N : \mathcal{Y}_N \to \mathbb{R}_+$ specifies the delay through driver circuit C to the source pin $p \in N$ ($\gamma(p) = C$). The delay depends linearly on the length of the Steiner tree $Y_N \in \mathcal{Y}_N$:

$$d_N(Y_N) := a_C + b_C \cdot \sum_{e \in E(Y_N)} l(e)$$

   with constants $a_C, b_C > 0$ depending on the driving circuit C.

   The delay bounds are preserved if

$$\sum_{N \in P \cap \mathcal{N}} d_N(Y_N) \leq D(P) \text{ for all } P \in \mathcal{P}.$$

   Show that the Simple Global Routing Problem with these additional delay constraints can be modeled as a Resource Sharing Problem. [3 points]

4. Modify Dijkstra's algorithm to prove the following statements.

   (a) Given a digraph $G = (V, E), s \in V$, and a length function $l : E \to \mathbb{Z}_+$, and an upper bound Δ on $\max\{dist_l(s, v) \mid v \text{ reachable from } s\}$, a shortest path tree rooted at s can be found in time $O(m + \Delta)$. [3 points]

   (b) Given a digraph $G = (V, E), s \in V$, and a length function $l : E \to \mathbb{Z}_+$, a shortest path tree rooted at s can be found in time $O(m \log_d L)$, where $d = \max\{2, m/n\}$, and $L := \max\{l(e) \mid e \in E\}$. [6 points]

The deadline for submitting solutions is July 21 before the lecture (12:15).