Chip Design
Summer term 2009

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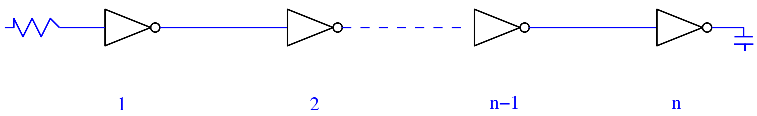
Exercises 10

1. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ $(1 \leq i \leq n)$. The $(i+1)$-th inverter is the successor of the i-th inverter for $1 \leq i < n$.



Ignore wire delays and assume the RC-delay model from the lecture, i.e. slews and transitions are ignored, and the delay θ_i through inverter i $(1 \leq i \leq n)$ is given by a posynomial

$$\theta_i(x) = \alpha + \frac{\beta}{x_i} C_i$$

where $x = (x_1, \ldots, x_n)$, $\alpha \geq 0$, $\beta > 0$, and $C_i = x_{i+1}$ for $i = 1, \ldots, n-1$. Furthermore, assume that the start time $\mathrm{at}(0, x)$ of the signal entering the first inverter (inverter 1) depends linearly on the inverter size ($\mathrm{at}(0, x) = \beta x_1$) and that the last inverter drives a fixed capacitance of $C_n \in \mathbb{R}_+$.

Derive a closed formula for the size x_i of the $i-th$ inverter in a solution x of the total delay minimization problem:

$$\min \left\{ \mathrm{at}(0, x) + \sum_{i=1}^{n} \theta_i(x) \mid x_i > 0 \text{ for all } 1 \leq i \leq n \right\}.$$

[6 points]

2. Prove Kraft's inequality (Proposition 6.1 in the lecture notes): Given a finite set S and predefined heights $a_s, s \in S$, there exists a binary tree with the elements of S as the leaves such that each $s \in S$ has height at most a_s if and only if

$$\sum_{s \in S} 2^{-a_s} \leq 1.$$

[5 points]

3. Consider the Repeater Tree Topology problem.

(a) Show that in case of integral a'_s $(s \in S)$ and $b = 1$,

$$\sigma_{opt} = -\left\lceil \log_2 \left(\sum_{s \in S} 2^{-a'_s} \right) \right\rceil .$$

[2 points]

(b) Show that variant (b) of the repeater tree topology algorithm does in general not result in a topology with optimum worst slack. [3 points]

(c) Show that a topology just maximizing the worst slack can be found in $O(|S| \log |S|)$ (Hint: Iteratively group the two most uncritical nodes). [4 points]

The deadline for submitting solutions is July 7 before the lecture (12:15).