1. Given a linear time-cost tradeoff instance \((G, T, c)\), show that a solution \(x\) is time-cost optimal if and only if \(C(x) = C_{\text{OPT}}(T(x))\). [5 points]

2. Given a discrete time-cost tradeoff instance \((G, T, c)\) and \(D, B \in \mathbb{R}_{\geq 0}\), the Discrete Time-Cost Tradeoff Decision Problem is to decide whether a solution \(x\) with \(T(x) \leq D\) and \(C(x) \leq B\) exists.
   Prove that the Discrete Time-Cost Tradeoff Decision Problem is \(\text{NP}\)-complete even if \(G\) is a path. (Hint: Recall the Knapsack Problem.) [3 points]

3. Recall the definition of a posynomial function and a geometric program on the back side.
   (a) Give an example for a non-convex posynomial function. [2 points]
   (b) Show that a geometric program can be transformed into an equivalent convex program by variable transformation \(y_i := \ln x_i\). (Hint: Use Hölder’s inequality.) [5 points]

4. Consider the soft (macro) circuit placement problem. Let \(C_1, C_2, \ldots, C_n\) be a set of circuits with area consumptions \(A_i \in \mathbb{R}\) and lower and upper bounds \(L_i, U_i \in \mathbb{R}_+\) for their widths \(w(C_i): L_i \leq w(C_i) \leq U_i \ (i \in \{1, \ldots, n\})\). Note, heights \(h(C_i)\) and widths \(w(C_i)\) are related by \(h(C_i) \cdot w(C_i) = A_i\).
   Furthermore, there is a sequence pair \((\pi, \rho)\) determining the relative positions of the circuits. Show that minimizing the size \(x_{\text{max}} y_{\text{max}}\) of the placement area \([0,0] \times [x_{\text{max}}, y_{\text{max}}]\) such that there are legal widths and heights for all circuits, and all circuits can be placed legally in \([0,0] \times [x_{\text{max}}, y_{\text{max}}]\) preserving the sequence-pair constraints can be solved by a smooth convex program. [4 points]

The deadline for submitting solutions is June 30 before the lecture (12:15).
**Definition** A posynomial function $f : \mathbb{R}_+^n \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}},$$

where $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$. Posynomials with $K = 1$ are called *monomials*. A geometric program is an optimization problem of the form

$$\begin{align*}
\min & \quad f_0(x) \\
\text{s.t.} & \quad f_i(x) \leq 1 \quad (i = 1, \ldots, n) \\
& \quad g_j(x) = 1 \quad (j = 1, \ldots, m)
\end{align*}$$

with posynomials $f_0, f_i : \mathbb{R}_+^n \to \mathbb{R}$ ($i = 1, \ldots, n$) and monomials $g_j : \mathbb{R}_+^n \to \mathbb{R}$ ($j = 1, \ldots, m$).