Prof. Dr. Jens Vygen
Dr. Stephan Held

Exercises 9

1. Given a linear time-cost tradeoff instance $(G, \mathcal{T}, c)$, show that a solution x is time-cost optimal if and only if $C(x) = C_{OPT}(T(x))$. [5 points]

2. Given a discrete time-cost tradeoff instance $(G, \mathcal{T}, c)$ and $D, B \in \mathbb{R}_{\geq 0}$, the Discrete Time-Cost Tradeoff Decision Problem is to decide whether a solution x with $T(x) \leq D$ and $C(x) \leq B$ exists.

   Prove that the Discrete Time-Cost Tradeoff Decision Problem is *NP*-complete even if G is a path. (Hint: Recall the Knapsack Problem.) [3 points]

3. Recall the definition of a posynimial function and a geometric program on the back side.

   (a) Give an example for a non-convex posynomial function. [2 points]

   (b) Show that a geometric program can be transformed into an equivalent convex program by variable transformation $y_i := \ln x_i$. (Hint: Use Hölder's inequality.) [5 points]

4. Consider the soft (macro) circuit placement problem. Let $C_1, C_2, \ldots, C_n$ be a set of circuits with area consumptions $A_i \in \mathbb{R}$ and lower and upper bounds $L_i, U_i \in \mathbb{R}_+$ for their widths $w(C_i)$: $L_i \leq w(C_i) \leq U_i$ $(i \in \{1, \ldots, n\})$. Note, heights $h(C_i)$ and widths $w(C_i)$ are related by $h(C_i) \cdot w(C_i) = A_i$.

   Furthermore, there is a sequence pair (π, ρ) determining the relative positions of the circuits. Show that minimizing the size $x_{\max} y_{\max}$ of the placement area $[0, 0] \times [x_{\max}, y_{\max}]$ such that a there are legal widths and heights for all circuits, and all circuits can be placed legally in $[0, 0] \times [x_{\max}, y_{\max}]$ preserving the sequence-pair constraints can be solved by a smooth convex program. [4 points]

The deadline for submitting solutions is June 30 before the lecture (12:15).

Definition A posynomial function $f : \mathbb{R}^n_+ \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}},$$

where $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$. Posynomials with $K = 1$ are called *monomials*. A geometric program is an optimizaiton problem of the form

$$\begin{array}{lllll} \min & f_0(x) & & & \\ \text{s.t.} & f_i(x) & \leq & 1 & (i = 1, \ldots, n) \\ & g_j(x) & = & 1 & (j = 1, \ldots, m) \end{array}$$

with posynomials $f_0, f_i : \mathbb{R}^n_+ \to \mathbb{R}$ $(i = 1, \ldots, n)$ and monomials $g_j : \mathbb{R}^n_+ \to \mathbb{R}$ $(j = 1, \ldots, m)$.