Chip Design
Summer term 2009

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Exercises 8

1. Prove Proposition 4.2: Let G be a digraph, $c : E(G) \to \mathbb{R}$, and $F \subseteq E(G)$ such that $(V(G), E(G) \setminus F, c)$ is conservative. Then

$$\begin{aligned} & \max\left\{\min\{c_\pi(e) \ : \ e \in F\} \ : \ \pi : V(G) \to \mathbb{R}, c_\pi(e) \geq 0 \text{ for } e \in E(G) \setminus F\right\} \\ = \ & \min\left\{\tfrac{c(E(C))}{|F \cap E(C)|} \ : \ C \text{ circuit in } G, F \cap E(C) \neq \emptyset\right\} \\ = \ & \max\{\lambda \ : \ c_\lambda \text{ is conservative}\}. \end{aligned}$$

[5 points]

2. Consider the linear Time-Cost Tradeoff Problem.
   (a) Show that the deadline version is the dual of a Minimum Cost Flow Problem. [3 points]
   (b) Use (a) to develop a combinatorial polynomial time algorithm for the budget version. [2 points]

3. Prove Proposition 5.1: Let $(G, ([a_e, b_e])_{e \in E(G)}, c)$ be an instance of the linear Time-Cost Tradeoff Problem. Let x be a time-cost optimal solution and let x' be a solution with $T(x') < T(x)$. Describe a feasible solution of the following linear program:

$$\begin{array}{lrcll} \min & \sum_{e \in E_x} \gamma_e y_e & & & \\ s.t. & y_e & \geq & 0 & (e \in E_x^b) \\ & y_e & \leq & 0 & (e \in E_x^a) \\ & \zeta_w & \leq & \zeta_v + y_e & (e = (v,w) \in E_x) \\ & \zeta_t - \zeta_s & = & 1, & \end{array}$$

where $E_x := \bigcup\{E(P) \ : \ P \in \mathcal{P}(G), \sum_{e \in E(P)} x_e = T(x)\}$ is the set of critical edges, and $E_x^a := \{e \in E_x \ : \ x_e = a_e\}$ and $E_x^b := \{e \in E_x \ : \ x_e = b_e\}$. [5 points]

The deadline for submitting solutions is June 23 before the lecture (12:15).